

Computational **Materials Theory and** **Methods**

Lecture 1: **Classical Molecular Dynamics**

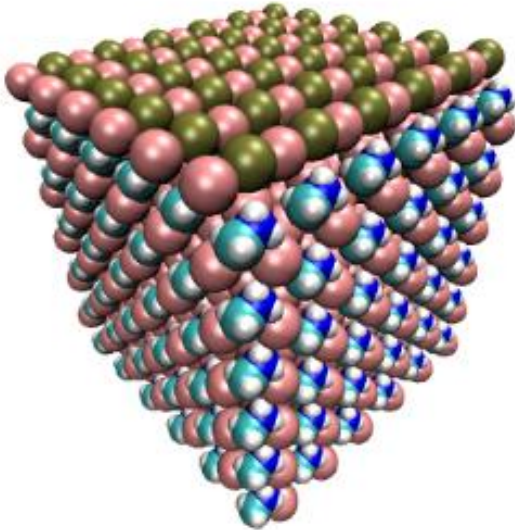
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Outline

- Modeling relies on statistical thermodynamics. Gibbs ensembles. Ensemble averages.
- Connection of statistical thermodynamics and MD. Ergodic hypothesis. Time and ensemble averages.
- Hamiltonian dynamics. EOMs and Integration schemes.
- Non-Hamiltonian dynamics. NVT ensemble
- Observables computed from MD simulations.
- MD modeling protocols
- Overview of available software for MD simulations.
- Demonstrations with Libra. Homework #1

Modeling material systems

Atomistic model



Properties

Equilibrium, static:

- Diffusion coefficients, D
- Heat capacity, C_v
- Caloric curves, phase transitions (E vs. T)
- Radial distribution function (RDF, $g(r)$)

Thermodynamic properties:

- Temperature, T
- Pressure, P
- Kinetic and potential energy, T and U
- Gibbs or Helholtz free energy, G and H

Structural characterization:

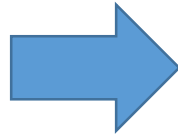
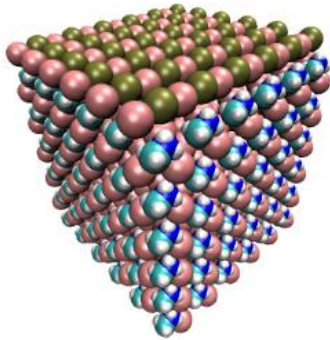
- Most stable structure

Dynamical properties:

- Kinetics of phase transitions
- Mechanisms of reactions
- Vibrational spectra, etc.

Modeling realistic systems

Ideal system



“Real” system

Real: all atoms interact with each other

Include all the non-idealities:

- Defects (vacancies, interstitials, dislocations, kinks, dangling bonds, etc.)
- Adsorbates (adatoms, surface dangling bonds passivation, etc.)
- Solvation and counterions
- Phases and grain boundaries

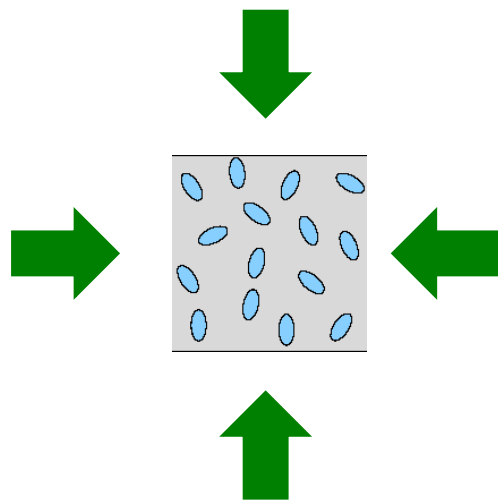
Periodic boundary conditions (PBC)

Real systems = are of N_A size, we can model $1 - 10^6$ atoms, usually around 1000

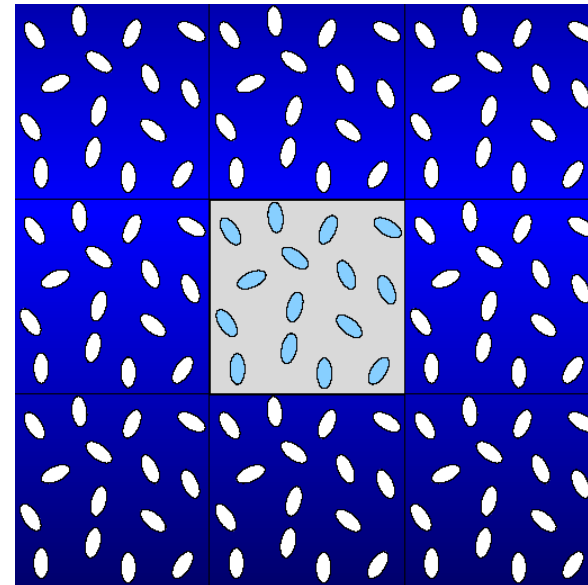
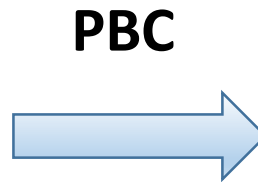
- Surface/volume ratio is large
- The structure of the surface is different from the bulk

To mitigate these effects:

Use PBC

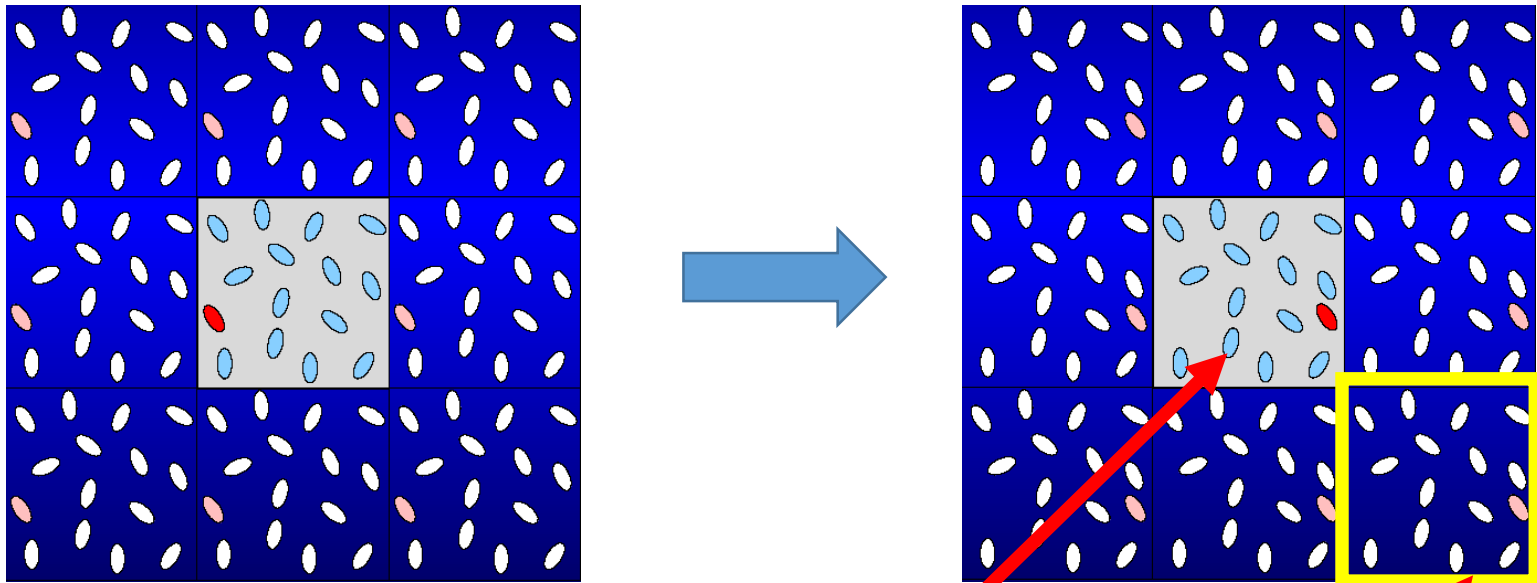


Surface defects!



How PBC works

Molecules that **exit from the left wall** **re-enter simulation cell from the right wall**



simulation cell

**replica
(periodic image)**

Gibbs Ensemble and phase spaces

Gibbs Ensemble

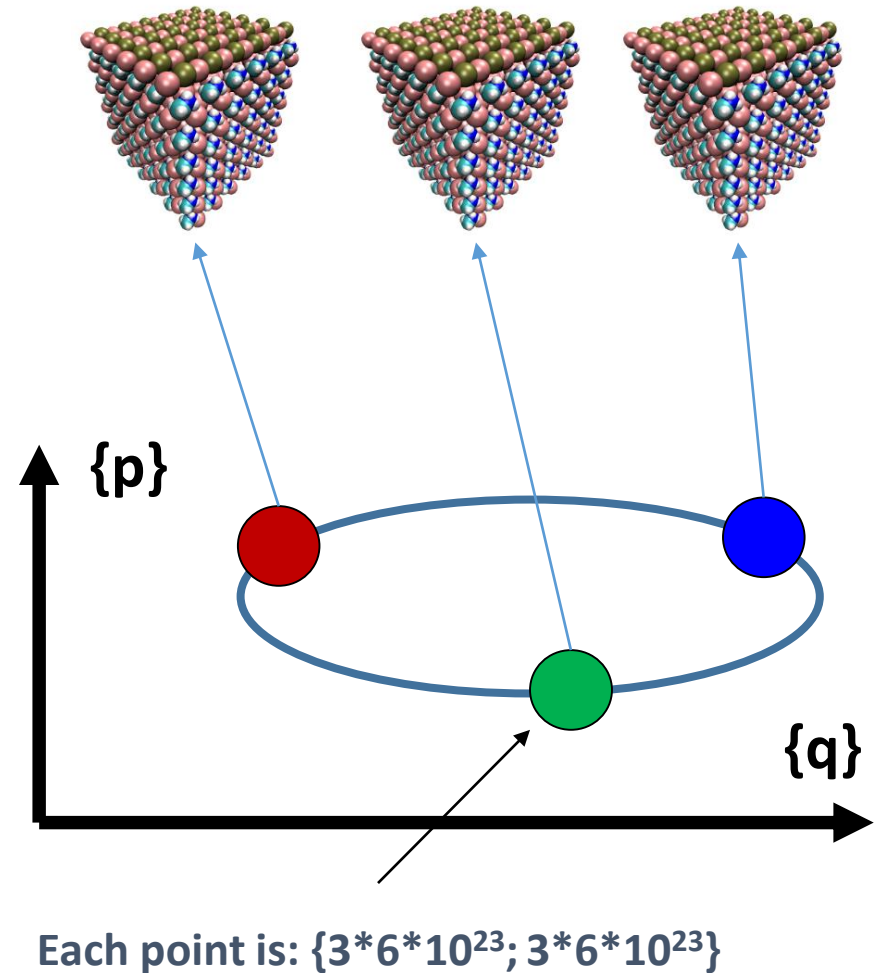
- Non-interacting copies (replicas) of the same “real” system.
- All copies corresponds to the same macroscopic properties (P, T, S, μ) but different microscopic properties (coordinates and velocities of atoms)

Phase Γ -space:

$$\Gamma = \{q, p\}$$

coordinates of a
1 mole of particles

momenta of a
1 mole of particles



Observed properties are statistical quantities

To compute properties, we compute thermal (ensemble) averages

$$A_{obs} = \langle A \rangle_T = \sum_{\Gamma_i} A(\Gamma_i) \omega(\Gamma_i)$$

$\omega(\Gamma_i)$ - a probability to find the system near the point Γ_i in the phase space

$A(\Gamma_i)$ - the value of the property of interest A at the point Γ_i in the phase space

$$A_{obs} = \langle A \rangle_T = \int_{\Gamma} A(\Gamma) d\omega(\Gamma)$$

$$A_{obs} = \langle A \rangle_T = \int_{\Gamma} A(\Gamma) \rho(\Gamma) d\Gamma$$

Ensemble average:

$$A_{obs} = \langle A \rangle_T = \frac{\int_{\Gamma} A(q, p) \rho(q, p) d^{3N}q d^{3N}p}{\int_{\Gamma} \rho(q, p) d^{3N}q d^{3N}p}$$

All properties are defined by the
probability distribution function, $\rho(\mathbf{q}, \mathbf{p})$!

Classification of the Gibbs ensembles

NVE (microcanonical): Constant number of particles (N), volume (V), and total energy (E)

$$\rho_{NVE}(\mathbf{q}, \mathbf{p}) = \delta(H(\mathbf{q}, \mathbf{p}) - E)$$

Nothing but the energy conservation requirement to the “regular” (Hamiltonian) dynamics

- If your integrator or system preparation are bad (don't conserve energy) – you do not sample points from the correct NVE distribution function!

NVT (canonical): Constant number of particles (N), volume (V), and temperature (T)

$$\rho_{NVT}(\mathbf{q}, \mathbf{p}) \sim \exp\left(-\frac{H(\mathbf{q}, \mathbf{p})}{k_B T}\right)$$

“Thermostatted” MD:

Nose, Nose-Hoover, Andersen thermostats

NPT (isobaric-isothermal): Constant number of particles (N), pressure (P), and temperature (T)

$$\rho_{NPT}(\mathbf{q}, \mathbf{p}) \sim \exp\left(-\frac{H(\mathbf{q}, \mathbf{p}) + PV}{k_B T}\right)$$

- Closer to real experimental conditions
- Critical to use in high-pressure studies

Examples of the Gibbs ensembles

$$H = \frac{p^2}{2m} + \frac{1}{2}kq^2$$

$$k = 0.1 \frac{Ha}{Bohr^2}; m = 100 a.u.; q_0 = 0.1 Bohr$$

Run for 25000 a.u.

NVE ensemble:

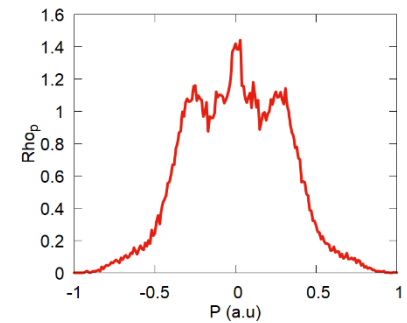
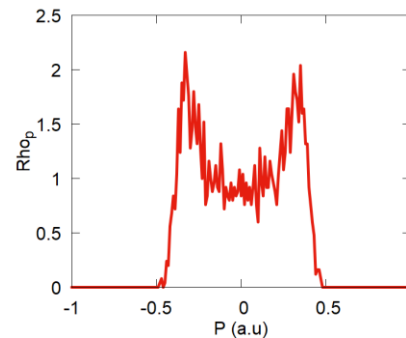
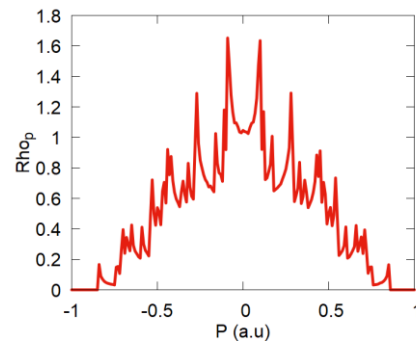
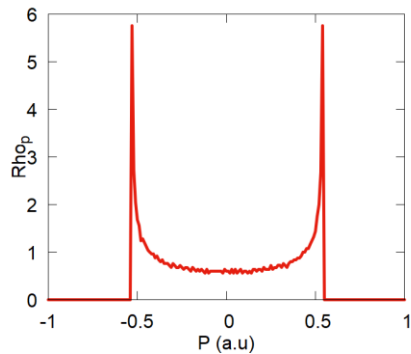
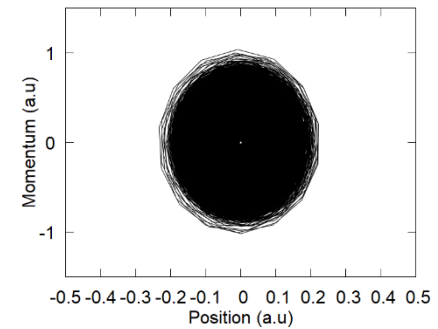
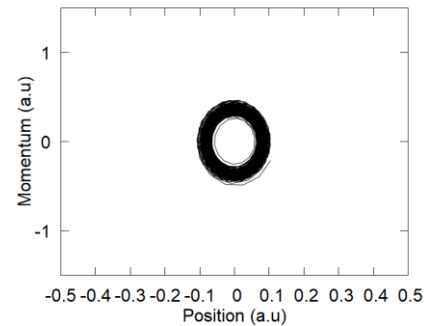
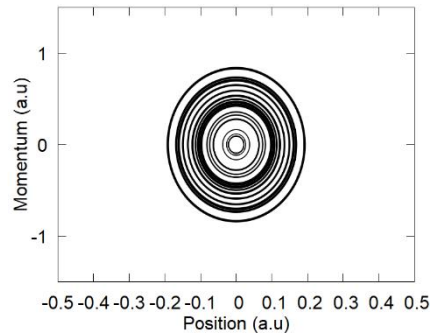
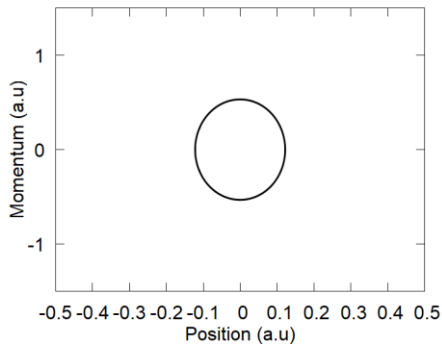
“NVT” ensemble:

Single trajectory

25 trajectories

Single trajectory

25 trajectories



Practical exercises with Libra

Define your potential: in “Hamiltonian.py”

```
obj.ham_dia.set(0,0, 0.5*k*x*x*(1.0+0.0j) )
```

$$H = \frac{1}{2} kx^2$$

```
obj.d1ham_dia[i].set(0,0, k*x*(1.0+0.0j) )
```

$$\frac{dH}{dx} = kx$$

Define your system: in e.g. “nvt.py”

```
nnucl, ntraj = 1, 25
```

How many DOFs is per 1 trajectory. How many trajectories

```
mean_q.set(0,0, 0.1)
```

The center of the initial distribution. Must specify for each DOF

```
iM.set(0,0, 1.0/100.0)
```

The masses of each DOF

Define the simulation parameters : in e.g. “nvt.py”

```
params["k"]
```

Harmonic force constant

```
params["dt"], params["nsteps"]
```

Integration time step (a.u.) and the number of steps

Run the MD : in e.g. “nvt.py”

```
Q, P = run_nvt(nnucl, ntraj, q, p, iM, compute_model, params)
```

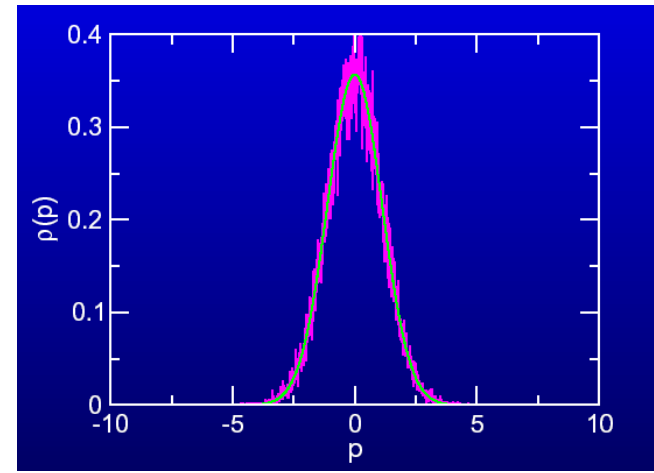
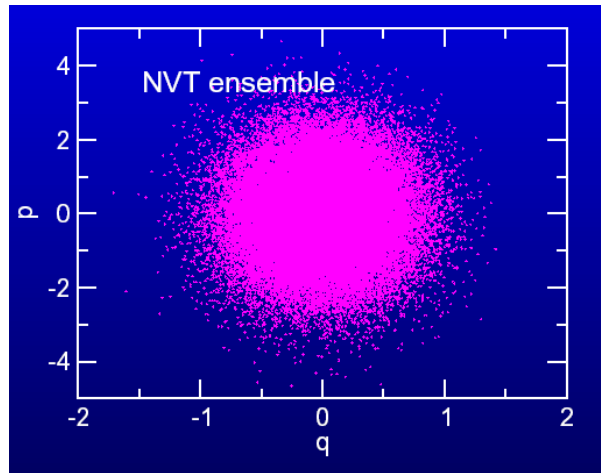
Run the analysis of MD trajectories : in e.g. “nvt.py”

```
compute_statistics(Q, idof, minx, maxx, dx, "_density_q.txt")
```

Examples of the Gibbs ensembles

Ideally, in the canonical ensemble we want:

$$\rho_{NVT} \sim \exp\left(-\frac{H(q, p)}{k_B T}\right) \quad d\omega(p_x) = \frac{1}{\sqrt{2\pi m k_B T}} \exp\left(-\frac{p_x^2}{2m k_B T}\right) dp_x$$



- This may not happen in reality: **Ergodicity**
- There are other methods of sampling (e.g. MC)

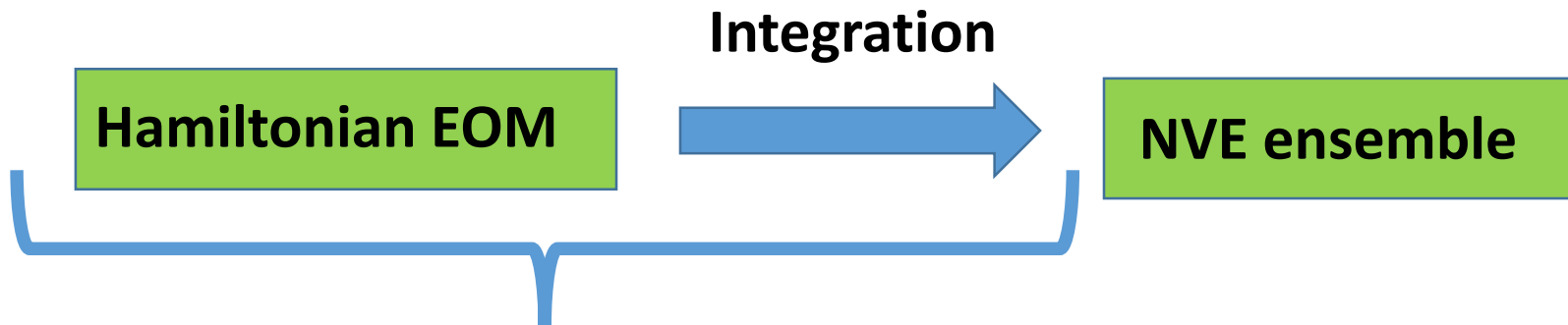
Molecular Dynamics : A way of getting ρ

This is where the statistical thermodynamics is connected to MD!!!

NVE ensemble

Evolution of a point in the phase Γ -space (system) is given by the integrating classical equations of motion (EOM).

The correct integration of the Hamiltonian EOMs ensures the Total energy conservation.



Hamiltonian dynamics

Connection between Stat. Mech. and MD

Ergodic hypothesis

Given an **infinite time** to evolve, the system will visit all the points of the **phase space**



$$A_{obs} = \langle A \rangle_{ens} = \langle A \rangle_{time} \equiv \frac{1}{N_T} \sum_{i=1}^{N_T} A(\Gamma(t_i)) = \frac{1}{T} \int_0^T A(\Gamma(t)) dt$$

The probability density is the distribution of the “sampled” points in the phase space (sampled using **method for a given ensemble**)

Dealing with the ergodicity

$$A_{obs} = \langle A \rangle_{ens} = \langle A \rangle_{time} \equiv \frac{1}{N_T} \sum_{i=1}^{N_T} A(\Gamma(t_i)) = \frac{1}{T} \int_0^T A(\Gamma(t)) dt$$

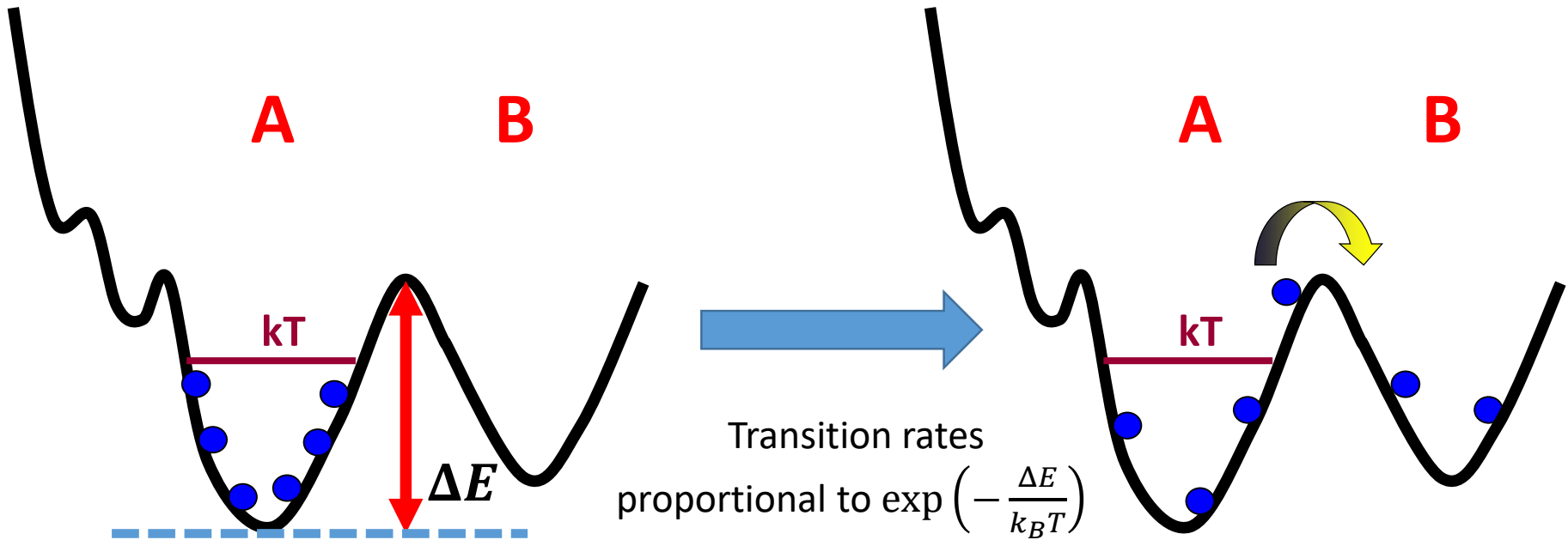
Problem: How to make sure this is true?

Criteria:

- Sufficiently long trajectories
- Starting with many points
- Choosing the right method

Criterion #1: Sufficiently long trajectories

A protein (as well as any material system) may have two dominant conformations: A and B



If we start in the conformation A, if the barrier ΔE is high, and temperature T is low, we'll have to run a **really long simulation** (beyond the computer capabilities maybe)

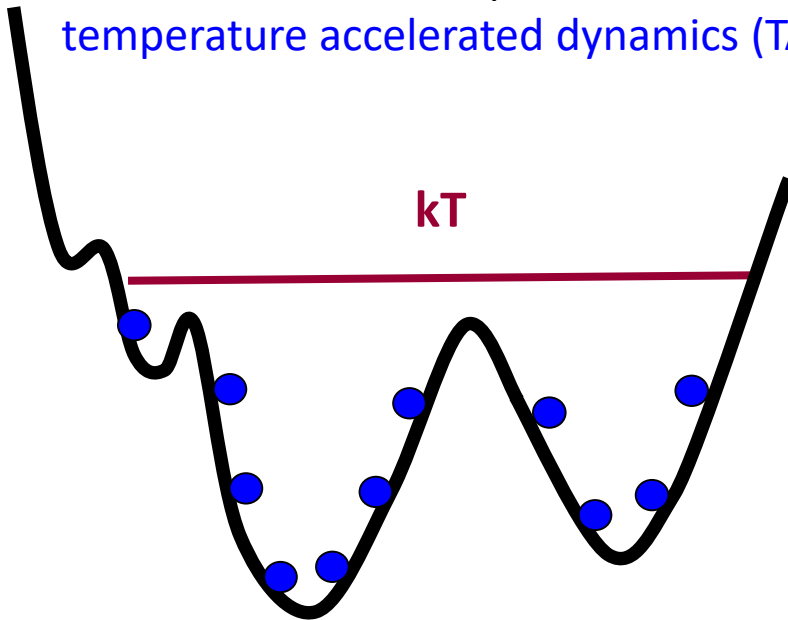
This is fine (to a certain extent!) to study the dynamics of transitions, but not to compute thermodynamic properties!

Criterion #1: Sufficiently long trajectories (cont.)

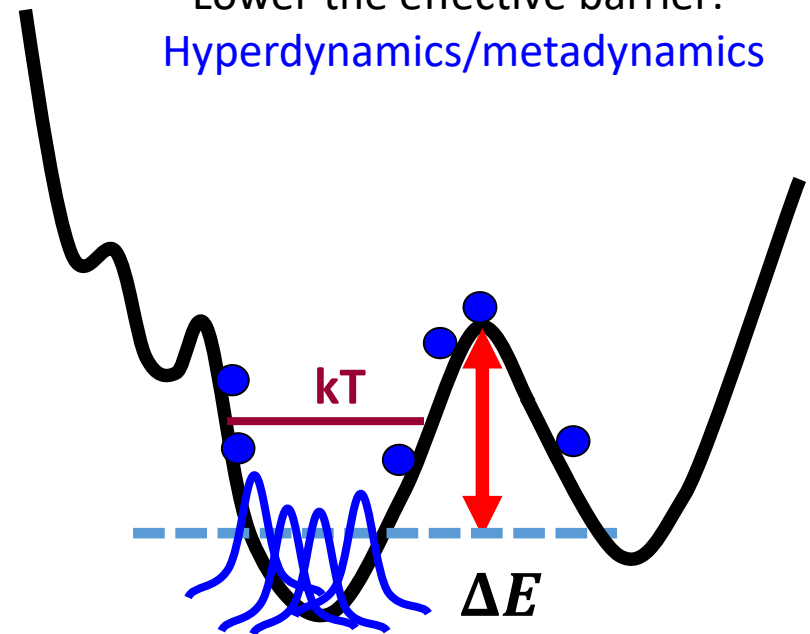
What can we do about it?

$$\exp\left(-\frac{\Delta E}{k_B T}\right)$$

Increase temperature:
temperature accelerated dynamics (TAD)



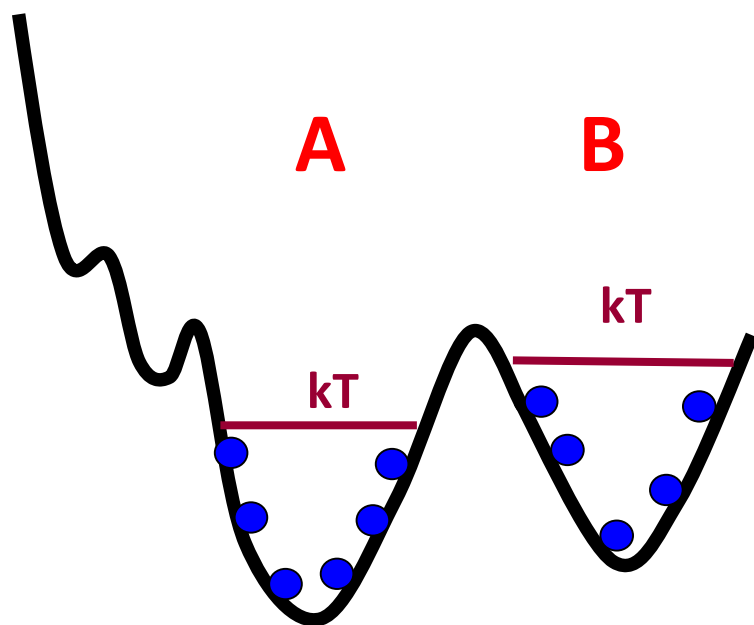
Lower the effective barrier:
Hyperdynamics/metadynamics



- The computed rates must be “rescaled” to the original temperature/barrier
- The dynamics is fictitious

Criterion #2: Starting points

How about if we initialize one fraction of the trajectories to be in the valley A, and the other fraction – to be in the valley B?



- We don't care about slow transitions
- Need to know the relevant regions in advance (e.g. chemical intuition)

Starting point refers not only to initial geometries!

Remember, we work with

$\{q, p\}$

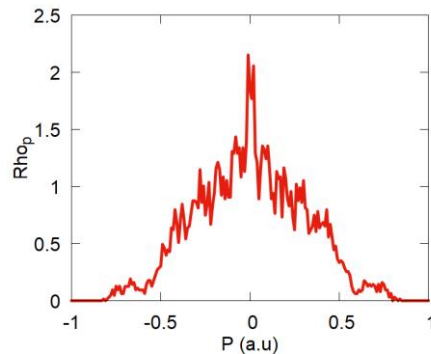
Initial geometries
(conformations)

Initial momenta

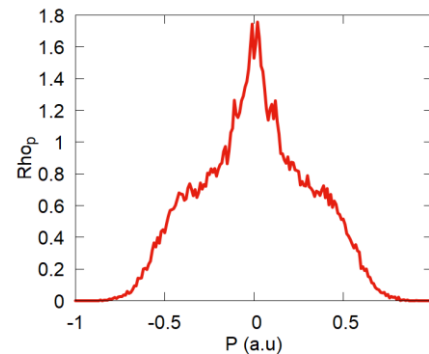
Criteria #1 and #2: Long trajectories & Starting points

25 trajectories

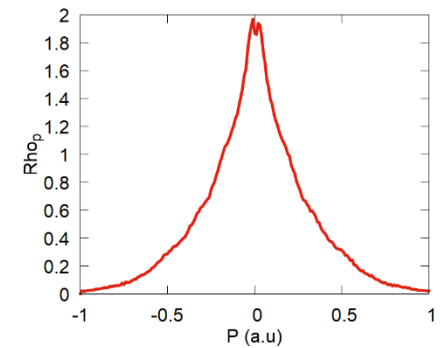
params["nsteps"] = 250



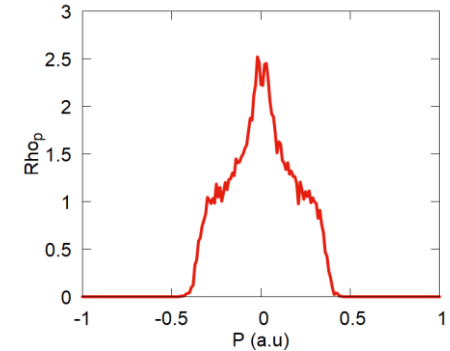
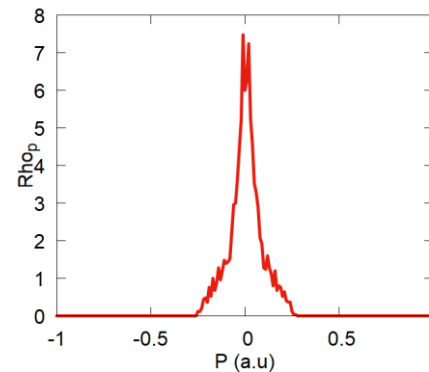
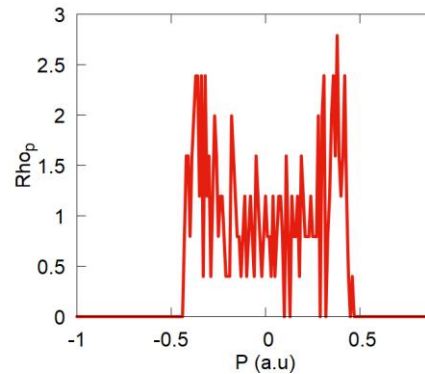
params["nsteps"] = 2500



params["nsteps"] = 25000



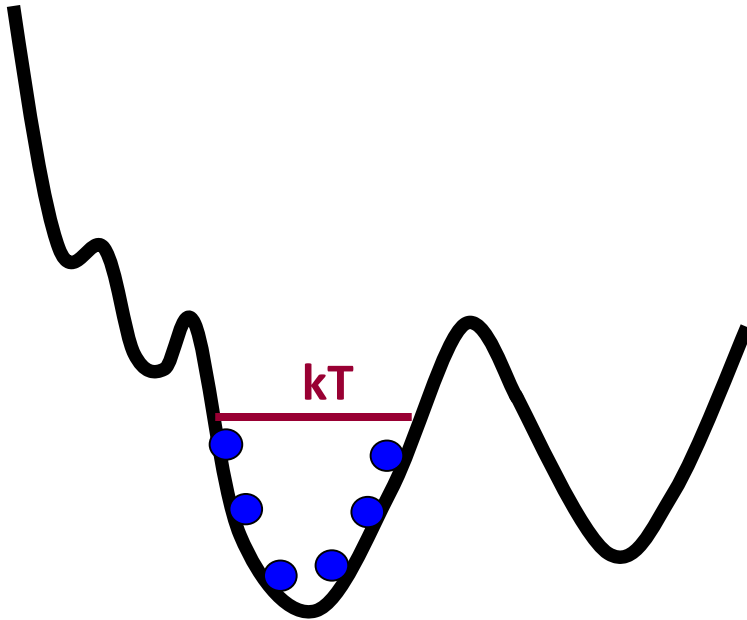
1 trajectory



25 trajectories (1 DOF each) coupled to a Nose-Hoover thermostat (“NVT”) ensemble

Criterion #3: Choosing the right method

Not all **systems** and not all **methods** lead to ergodicity



In the NVE ensemble (Hamiltonian dynamics):
you will stay in this valley forever!

Energy conservation!

Thermodynamic limit ($N \rightarrow \infty$):
NVE and NVT ensembles are equivalent

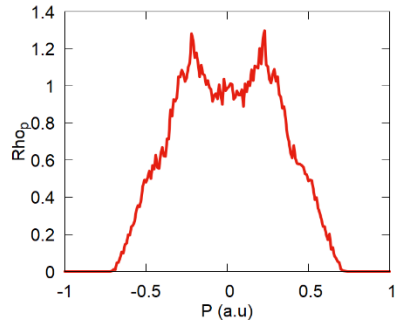
Smaller systems (e.g. N dof = 1): they are
notably different

To explore other valleys, we need to
utilize NVT simulations!
(non-Hamiltonian dynamics)

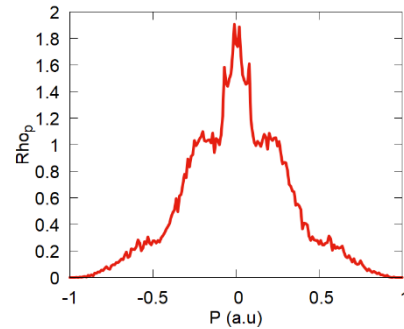
The total energy of the **system**
is not conserved!

Criterion #3: Using the right method

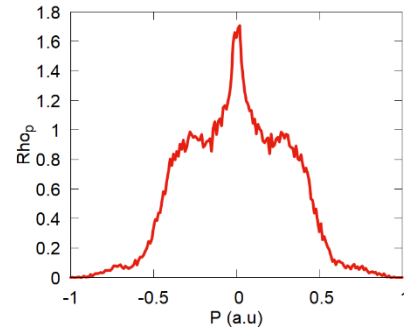
Chain size = 1



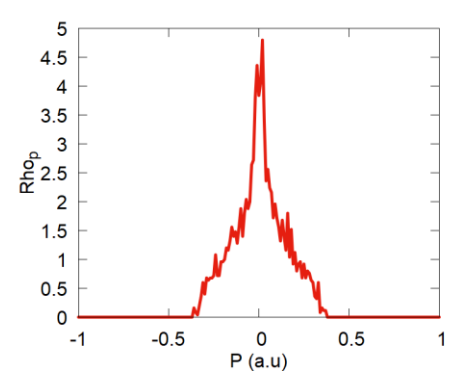
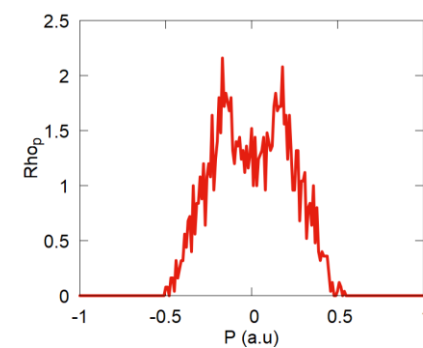
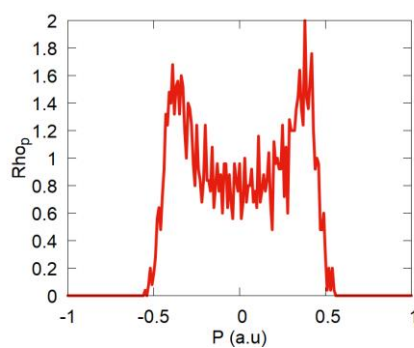
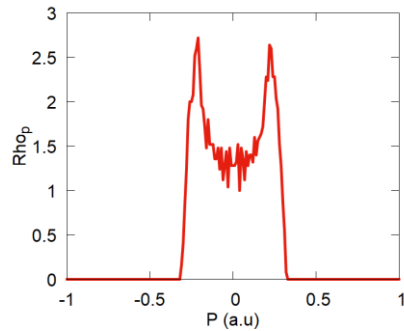
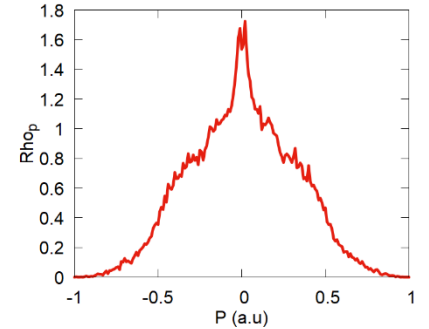
Chain size = 5



Chain size = 25



Chain size = 250

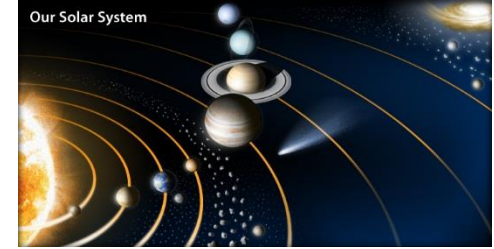


This is not quite the right method: the NVT ensemble doesn't make sense for a system of 1 nuclear DOF

Basic terminology of classical mechanics

Material point: neglect size

What is large? Is the Earth large? Compared to what?



Choice of the coordinates:

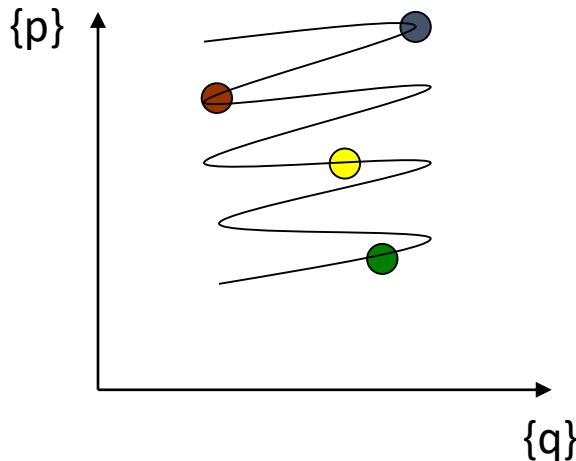
- Cartesian or Internal
- Can be chosen as convenient

Equations of motion(EOM):

$$f(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}) = 0$$

This is what we solve by doing MD

Trajectory:



$$\mathbf{r}(t) = (x_1(t), y_1(t), z_1(t), x_2(t), y_2(t), z_2(t), \dots)^T$$

$$\mathbf{r}(t) = (\mathbf{r}_1(t), \mathbf{r}_2(t), \dots, \mathbf{r}_N(t))^T$$

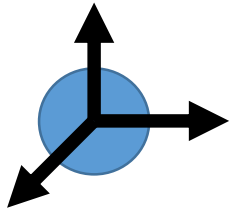
$$\mathbf{r}_1(t) = (x_1(t), y_1(t), z_1(t))^T = \begin{pmatrix} x_1(t) \\ y_1(t) \\ z_1(t) \end{pmatrix}$$

Degrees of freedom (DOF)

DOF = parameters that fully specify the geometry of a system

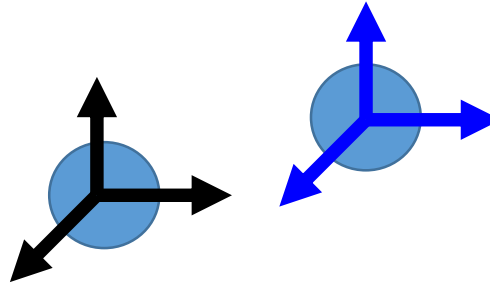
The number of DOFs: $3N - (\# \text{ of constraints})$

An atom: $3 \times 1 = 3$ DOFs

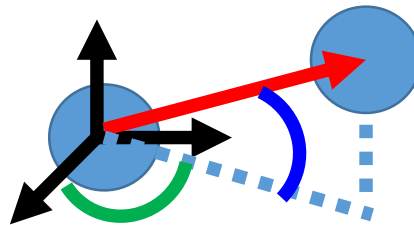


DOFs: x, y, z

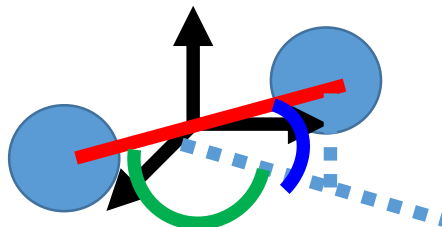
Diatomic molecule: $3 \times 2 = 6$ DOFs



DOFs: $x_1, y_1, z_1, x_2, y_2, z_2$
(Cartesian)



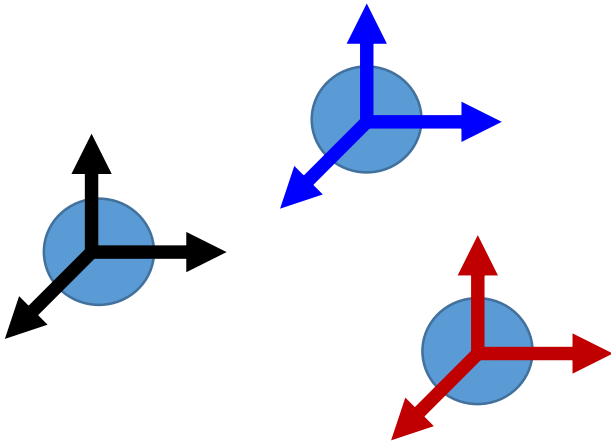
DOFs: $x_1, y_1, z_1, r, \phi, \theta$ (polar)



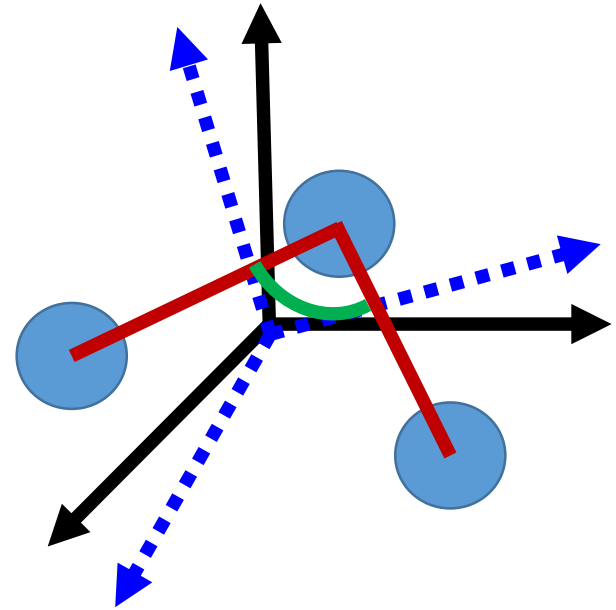
DOFs: X, Y, Z, r, ϕ, θ (internal)

Degrees of freedom (DOF)

Triatomic molecule: $3 \times 3 = 9$ DOFs



DOFs: $x_1, y_1, z_1, x_2, y_2, z_2, x_3, y_3, z_3$
(Cartesian)



DOFs (internal):

X, Y, Z – position of the center of mass (COM)

Θ, ϕ, ψ – angles the molecular inertia axes form with the external (Cartesian) coordinate system

r_{12}, r_{23} – interatomic distances

Θ_{123} – an angle between two bonds

Hamiltonian Dynamics

$$H = K + U = \sum_{i=1}^N \frac{\mathbf{p}_i^2}{2m_i} + U(\{\mathbf{q}\}) = \sum_{i=1}^N \frac{p_{x,i}^2 + p_{y,i}^2 + p_{z,i}^2}{2m_i} + U(\{q_1, q_2, \dots, q_{3N}\})$$

Total energy
(Hamiltonian)

Kinetic energy

Potential energy

Hamiltonian EOM

(generate an NVE ensemble)

$$\dot{q}_i = \frac{\partial H}{\partial p_i}$$
$$\dot{p}_i = -\frac{\partial H}{\partial q_i}$$

In a nutshell: **Newton's second law**

$$v_i = \dot{q}_i = \frac{p_i}{m_i}$$

$$\dot{p}_i = m_i a_i = -\frac{\partial U}{\partial q_i} \equiv F_i$$

Forces in MD

$$\mathbf{F} = -\frac{\partial U}{\partial \mathbf{q}}$$

what this really means: we compute the x,y, and z components of the forces acting on all particles i just by taking derivatives of the potential energy w.r.t. the corresponding coordinate

More explicitly:

$$\begin{aligned} F_{1,x} &= -\frac{\partial U}{\partial x_1} & F_{1,y} &= -\frac{\partial U}{\partial y_1} & F_{1,z} &= -\frac{\partial U}{\partial z_1} \\ F_{2,x} &= -\frac{\partial U}{\partial x_2} & F_{2,y} &= -\frac{\partial U}{\partial y_2} & F_{2,z} &= -\frac{\partial U}{\partial z_2} \\ & & \dots & & & \\ F_{N,x} &= -\frac{\partial U}{\partial x_N} & F_{N,y} &= -\frac{\partial U}{\partial y_N} & F_{N,z} &= -\frac{\partial U}{\partial z_N} \end{aligned}$$

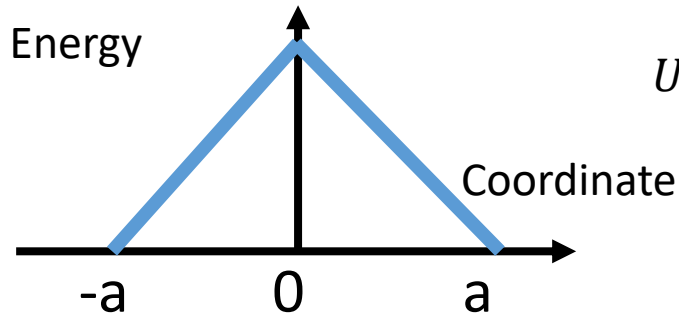
Force and energy evaluations – are the most expensive (time consuming) parts of MD simulations

Where to get U?

- Model Problems (here)
- Force Fields (next)
- Quantum Mechanics (later)

Forces in MD

Forces must be continuous! Or the MD won't be stable (will produce meaningless data)



$$U = \begin{cases} k(x + a), & x \in [-a, 0] \\ k(a - x), & x \in [0, a] \end{cases} \longrightarrow F = \begin{cases} -k, & x \in [-a, 0] \\ k, & x \in [0, a] \end{cases}$$

This is gonna be a problematic potential!

Don't forget the sign! Otherwise the system will blow up

Defined in Hamiltonian.py

```
def compute_model(q, params, full_id):  
    H = 1/2 * k * x**2    obj.ham_dia.set(0,0, 0.5*k*x*x*(1.0+0.0j) )  
    dH/dx = kx          obj.d1ham_dia[i].set(0,0, k*x*(1.0+0.0j) )
```

Beware! Here we compute just the derivative, not the force, which is a negative derivative!

Exercises:

- Can you define your Python function for Morse potential?
- Can you define your Harmonic potential for 2 particles connected by a spring in 1 D?

Solving (integrating) the Hamiltonian EOM

Taylor series

$$q_i(t + dt) \approx q_i(t) + \frac{p_i(t)}{m_i} dt + \frac{f_i(t)}{2m_i} dt^2 + O(dt^3) + O(dt^4)$$

$$q_i(t - dt) \approx q_i(t) - \frac{p_i(t)}{m_i} dt + \frac{f_i(t)}{2m_i} dt^2 - O(dt^3) + O(dt^4)$$

$$q_i(t + dt) \approx 2q_i(t) - q_i(t - dt) + \frac{f_i(t)}{m_i} dt^2 + O(dt^4)$$

$$p_i(t) \approx \frac{q_i(t + dt) - q_i(t - dt)}{2dt} + O(dt^3)$$

} Verlet algorithm

$$q_i(t + dt) \approx q_i(t) + p_i(t)dt + \frac{dt^2}{2} f_i(t)$$

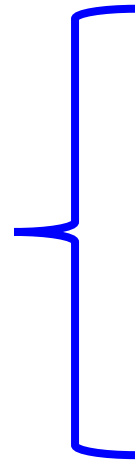
$$p_i(t + dt) \approx p_i(t) + \frac{dt}{2} [f_i(t) + f_i(t + dt)]$$

} velocity Verlet algorithm
More stable!

How to assess the quality of integration scheme?

In the isolated system (NVE),
the following quantities are conserved:

Invariants (integrals)
of motion:


$$\begin{aligned} H &= T + U && \text{total energy} \\ P &= \sum_i^N p_i && \text{total momentum} \\ L &= \sum_i^N l_i = \sum_i^N r_i \times p_i && \text{total angular momentum} \end{aligned}$$

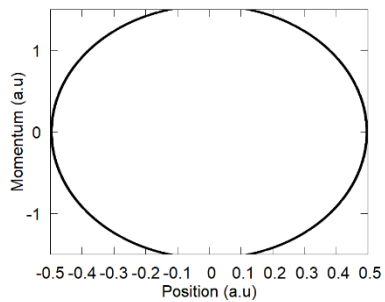
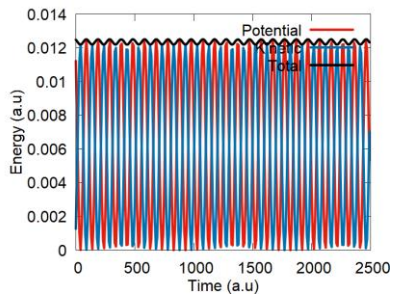
The energy conservation can be affected by:

- Integration time step vs. highest frequency $\omega = \sqrt{\frac{k}{m}}$
- Integration algorithm (e.g. Verlet vs. velocity Verlet)
- Force discontinuities
- Total energy of the systems (too “hot” requires smaller dt)

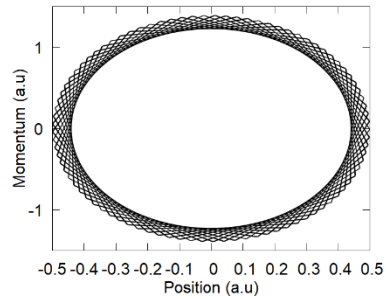
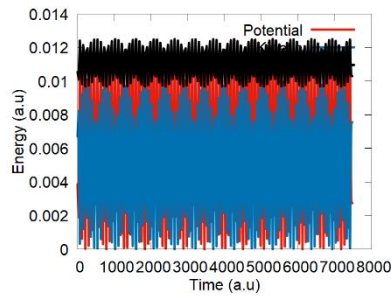
Illustration

$$k = 0.1, m = 100 \Rightarrow \omega = \sqrt{\frac{0.1}{100}} \approx 0.032 \text{ a.u.}^{-1} \Rightarrow dt \sim \frac{1}{0.032} = 31.6$$

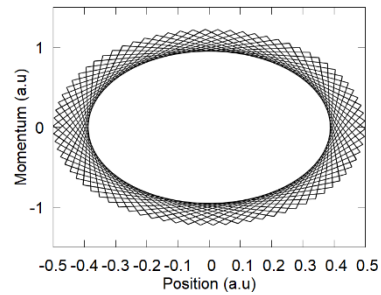
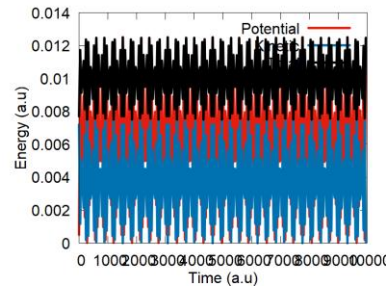
dt = 10



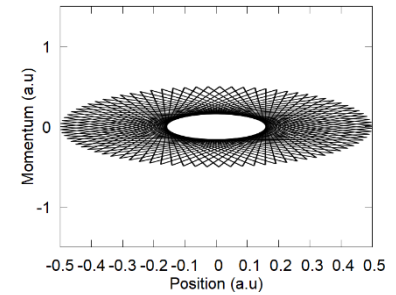
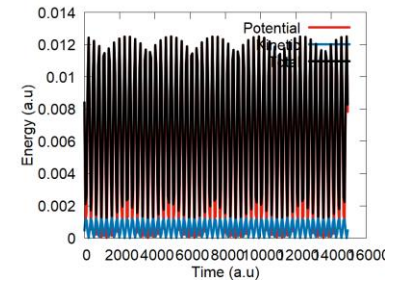
dt = 30



dt = 40



dt = 60



Non-Hamiltonian Dynamics

To generate NVT (and other ensembles) we can:

- rescale velocity to satisfy the temperature T ; → **incorrect sampling of fluctuations**
- add a random force and add a friction (Langevin/Andersen thermostat)
- rescale velocities by a factor not too different from 1.0, but which will eventually lead to the desired average temperature (Berendsen)
- Introduce extended phase space variables such that the integration over extra variables will yield the desired distribution (Nose, Nose-Hoover/chain thermostats, etc.)

$$\delta(H(q, p, s, p_s) - E) \rightarrow \int ds dp_s \delta(H(q, p, s, p_s) - E) = \exp\left(-\frac{H(q, p)}{k_B T}\right)$$

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \quad \dot{p}_i = -\frac{\partial H}{\partial q_i} - \xi p_i$$

There is **no Hamiltonian** from which these equations can be derived.

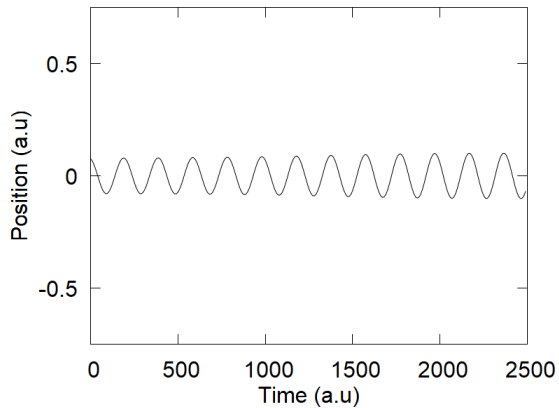
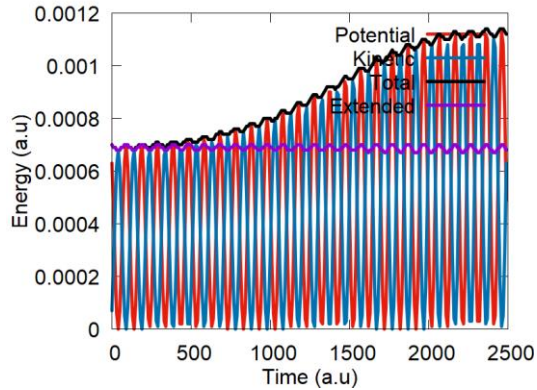
But, there is an extended energy, which is conserved along the motion.

non-Hamiltonian dynamics

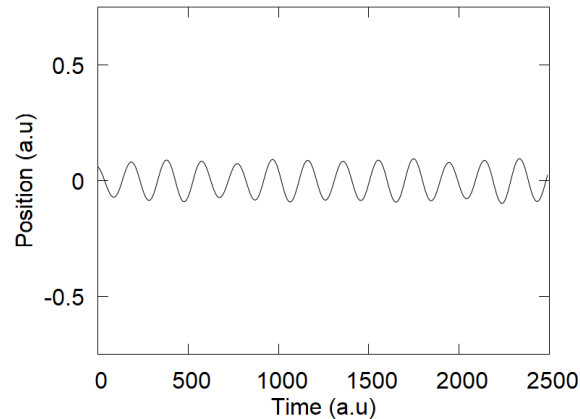
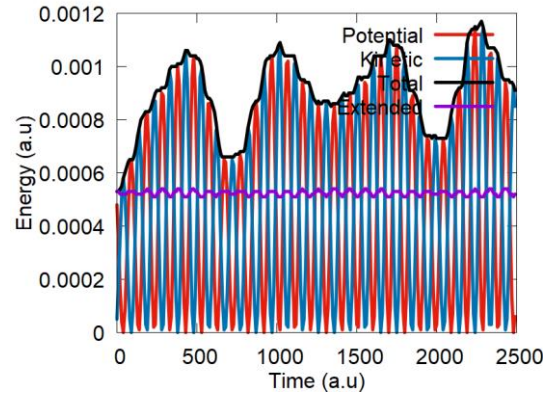
An illustration of Non-Hamiltonian Dynamics

Frequency of system-bath interaction

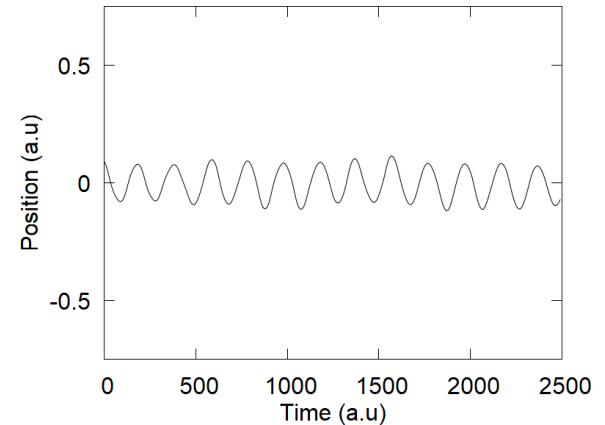
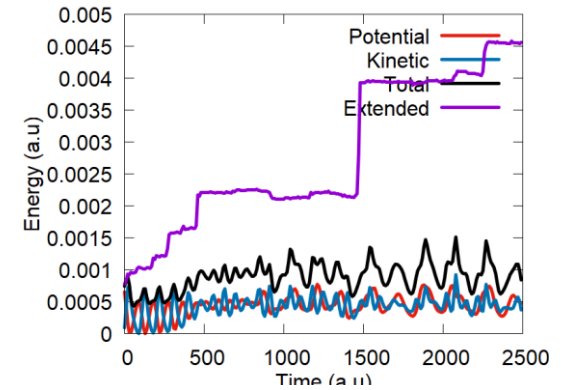
$\nu = 0.001$



$\nu = 0.01$



$\nu = 0.05$



← NVE limit

unstable →

Observables 1: Initial velocities and Temperature

$$\rho(v_x)dv_x = \frac{dN_{v_x}}{N} = \left(\frac{m}{2\pi k_B T}\right)^{1/2} \exp\left(-\frac{mv_x^2}{2k_B T}\right) dv_x$$

$$\rho(v)dv = \frac{dN_v}{N} = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2k_B T}\right) dv$$

Average velocity (magnitude):

$$\langle v \rangle = \int_0^\infty v \rho(v) dv = \sqrt{\frac{8k_B T}{\pi m}}$$

Average squared velocity:

$$\langle v^2 \rangle = \int_0^\infty v^2 \rho(v) dv = \frac{3k_B T}{m}$$

Kinetic energy:

$$K = \frac{1}{2} \sum_{i=1}^N m_i v_i^2$$

Average kinetic energy:

$$\langle K \rangle = \frac{3Nk_B T}{2}$$

Instantaneous temperature:

$$T = \frac{\sum_{i=1}^N m_i v_i^2}{3Nk_B}$$

Equipartition Principle: in classical limit, the average energy corresponding to any quadratic term in Hamiltonian is $\frac{1}{2} k_B T$

Observables 2: Pressure

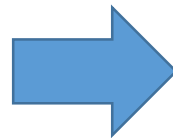
Virial theorem gives:

$$\langle K \rangle = -\frac{1}{2} \sum_{i=1}^N \langle q_i F_i \rangle$$

from which, one can find:

$$PV = Nk_B T - \frac{1}{3} \sum_{i=1}^N \left\langle q_i \frac{\partial U}{\partial q_i} \right\rangle$$

If the particles do not interact ($U = 0$):



$$PV = Nk_B T$$

the ideal gas law

Observables 3: Thermodynamics fluctuations

Within the canonic (NVT) ensemble

$$\langle \delta E^2 \rangle = k_B T^2 C_V$$



Heat capacity and phase transitions

$$\langle \delta K^2 \rangle = \frac{3}{2} N (k_B T)^2$$

$$\langle \delta U^2 \rangle = k_B T^2 \left(C_V - \frac{3Nk_B}{2} \right)$$

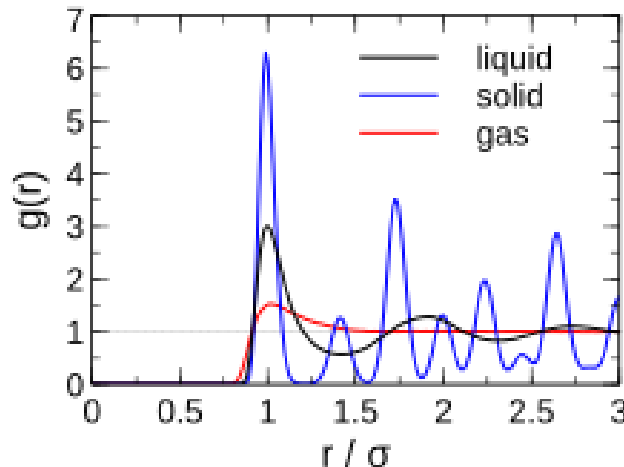
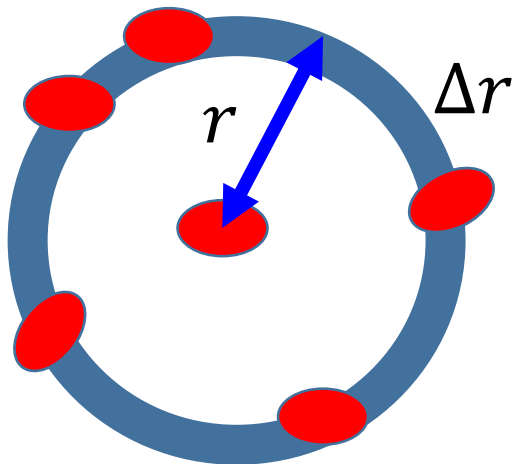
Observables 4: Radial distribution function (RDF)

- **RDF, $g(r)$:**

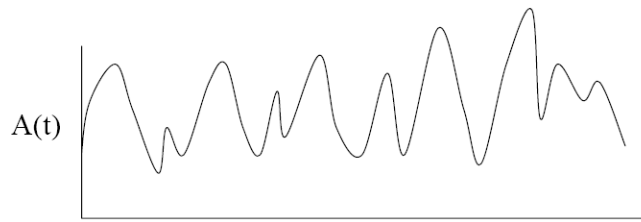
The probability to find two atoms at a given distance r from each other in comparison to the probability of the same but in an uniformly distributed system of the same density

$$g(r) = \frac{\rho(r)}{\langle \rho \rangle} = \frac{VN(r)}{4\pi r^2 \Delta r N}$$

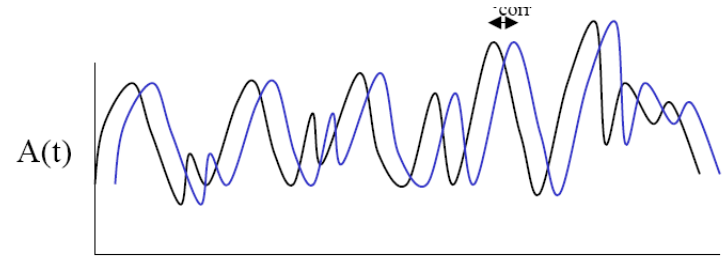
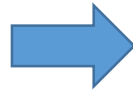
← The average number of particles that are within the shell $[r, r + \Delta r]$ from any other particle



Observables 5: Time-correlation function



Time, t



Time, t

$$C_{AB}(\tau) = \langle A(t)B(t + \tau) \rangle_{ens}$$

Diagram illustrating the calculation of the time-correlation function $\langle A(\Delta t)A(0) \rangle$ for different time intervals Δt .

The diagram shows three horizontal timelines representing time steps $A(t_0)$ through $A(t_9)$. The correlation function is calculated as the average of the product of values at time t and time $t + \Delta t$.

Top Panel (Grey background): Shows the correlation function $\langle A(\Delta t)A(0) \rangle$. The product terms are $A_0A_1 + A_1A_2 + A_2A_3 + A_3A_4 + A_4A_5 + A_5A_6 + A_6A_7 + A_7A_8 + A_8A_9 + \dots$. Green arrows connect $A(t_i)$ to $A(t_{i+1})$.

Middle Panel (Green background): Shows the correlation function $\langle A(2\Delta t)A(0) \rangle$. The product terms are $A_0A_2 + A_1A_3 + A_2A_4 + A_3A_5 + A_4A_6 + A_5A_7 + A_6A_8 + A_7A_9 + \dots$. Green arrows connect $A(t_i)$ to $A(t_{i+2})$.

Bottom Panel (Grey background): Shows the correlation function $\langle A(5\Delta t)A(0) \rangle$. The product terms are $A_0A_5 + \dots + A_3A_8 + A_4A_9 + \dots$. Green arrows connect $A(t_i)$ to $A(t_{i+5})$.

Observables 5: Velocity autocorrelation function

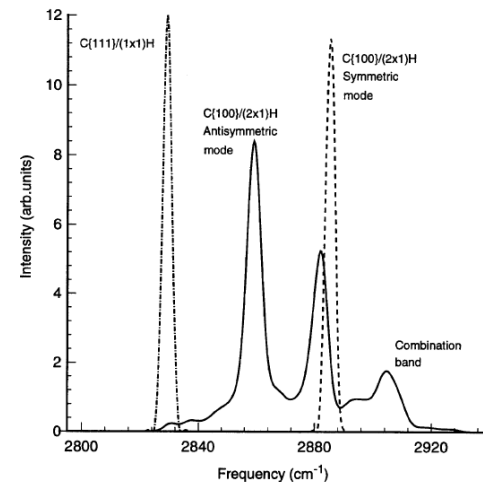
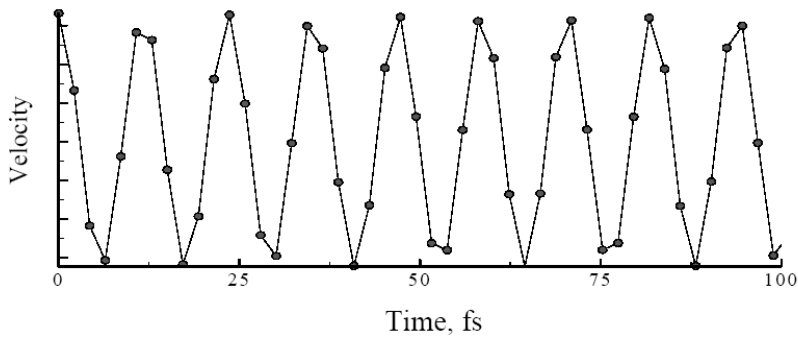
$$C_{vv}(\tau) = \langle v(t)v(t + \tau) \rangle_{ens} = \frac{1}{N} \sum_{i=1}^N \frac{1}{N_{in}} \sum_{t_0=1}^{N_{in}} v_i(t_0)v_i(t_0 + \tau)$$

Averaging is done w.r.t. the number of **particles** and w.r.t. the **initial times**

Optical response theory:

Fourier transform of the ACF gives an IR spectrum

$$I_{vv}(\omega) = \int_{-\infty}^{\infty} \exp(-i\omega\tau) C_{vv}(\tau) d\tau$$



Observables 6: Transport properties

$$\langle |q(t) - q(0)|^2 \rangle = 6Dt$$

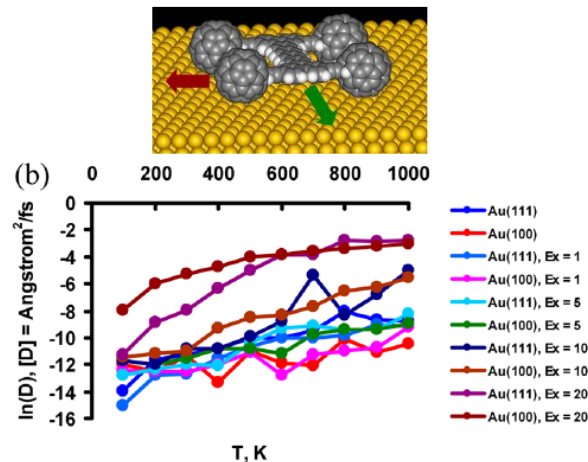
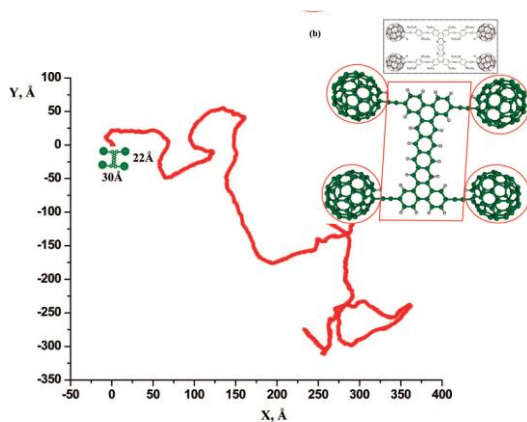
Einstein formula for diffusion coefficient in 3D

$$D = \int_0^\infty C_{vv}(\tau) d\tau$$

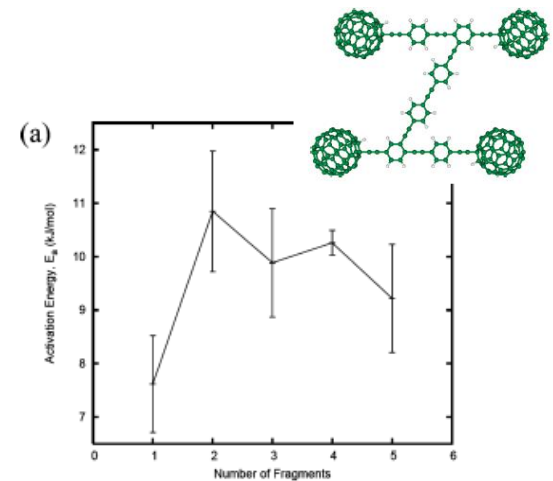
Green-Kubo formula

- Good with PBC
- Careful if stochastic thermostat

Activation energy for diffusion: $D \sim \exp(-\Delta E_a/k_B T)$



J. Phys. Chem. C **2012**,
116, 22595–22601.



J. Chem. Theory Comput.
2010, 6, 2581–2590.

J. Chem. Theory Comput.
2008, 4, 652–656.

General MD algorithm

Initial conditions:

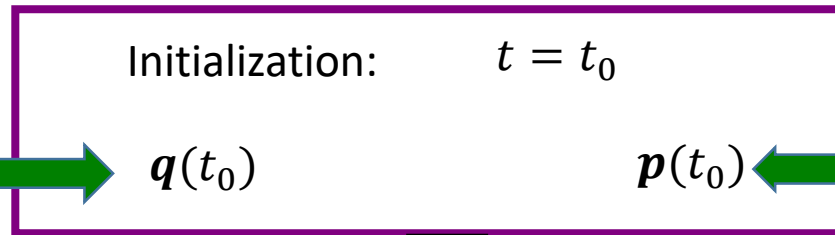
Input structure -
e.g. from a database,
guess structure

*Describe the chemistry
of the system*

Initial conditions:

E.g. sample from
the Maxwell-Boltzmann
distribution using MC.

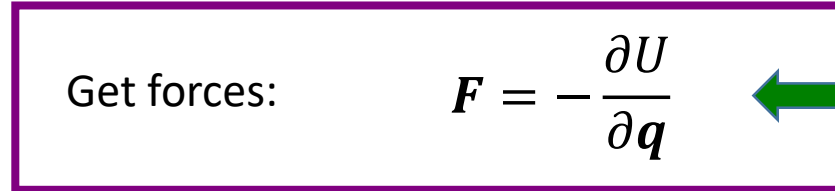
Describe the conditions



Interactions:

QM,
Force fields,
Models

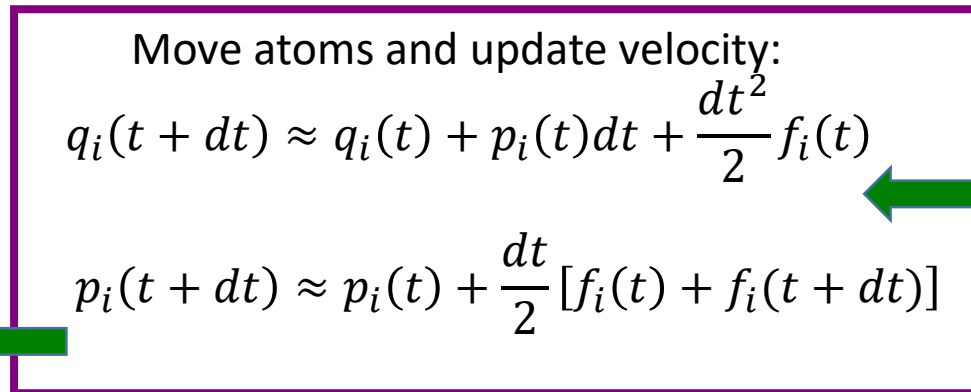
*Describe the physics
of the system*



Integration:

Equations to
sample
NVE or NVT
Ensemble

*Describe the
statistics
of the system*



$t = t + dt$

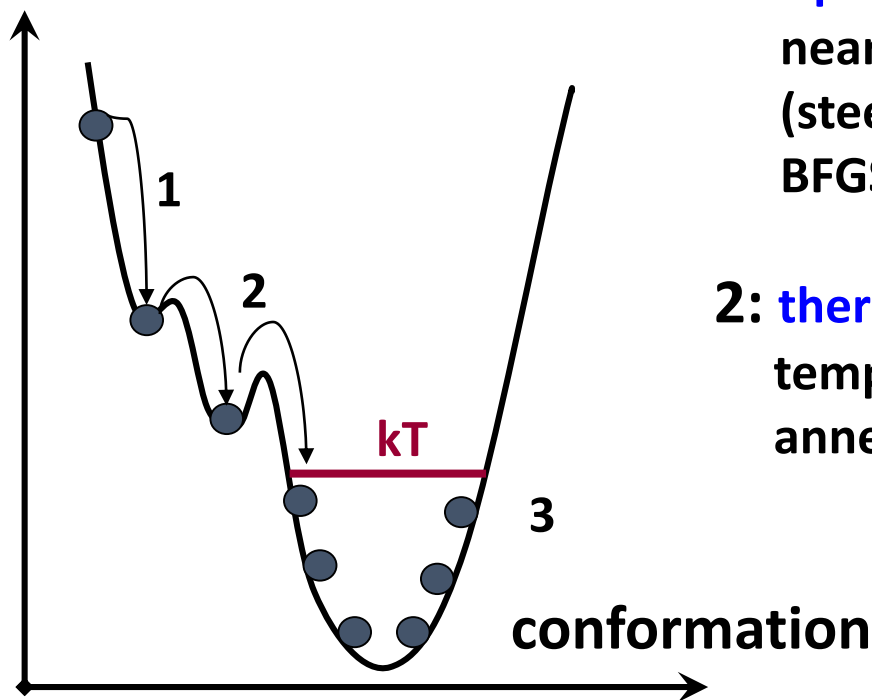
Analysis:

Compute the
Desired properties.

*Connection to
experiment.*

Preparation to MD simulations

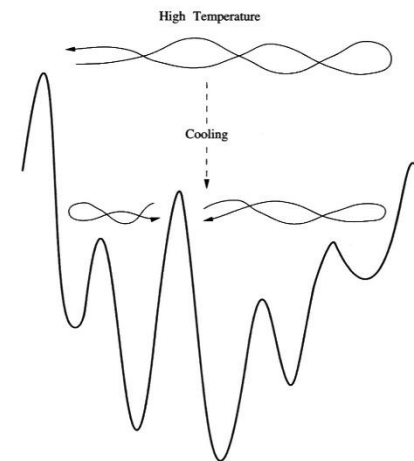
Energy



1: optimize your structure to the nearby (local) energy minimum (steepest descent, Newton's method, BFGS, etc.)

2: thermalize your structure to target temperature (NVT ensemble MD, simulated annealing, etc.)

3: production run. Sample conformation from desired distribution using suitable MD or MC algorithm

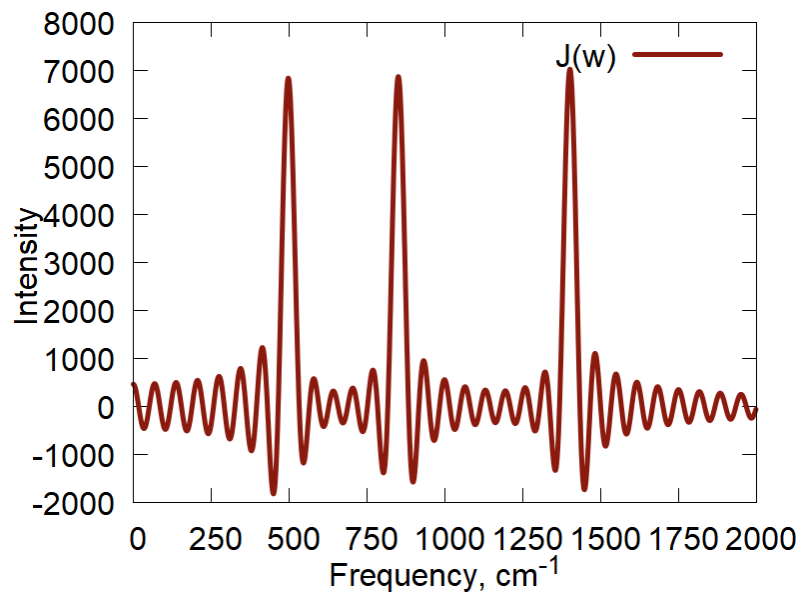
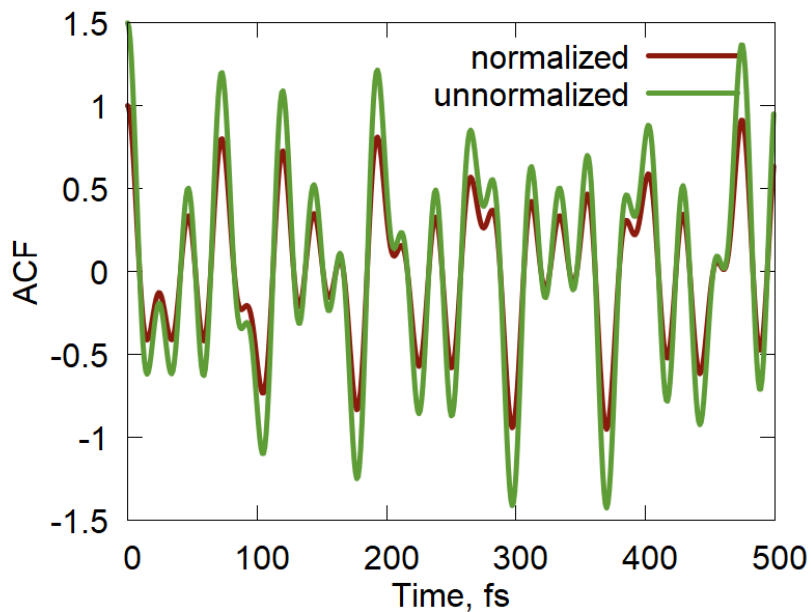


Exercises: ACF and Spectrum of a predefined data

In Tut2

$$r(t) = (\sin(\omega_1 t), \cos(\omega_2 t), \sin(\omega_3 t))$$

$$\begin{aligned}\omega_1 &= 500 \text{ cm}^{-1} \\ \omega_2 &= 1400 \text{ cm}^{-1} \\ \omega_3 &= 850 \text{ cm}^{-1}\end{aligned}$$



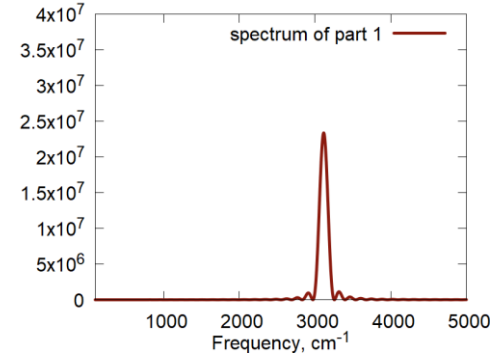
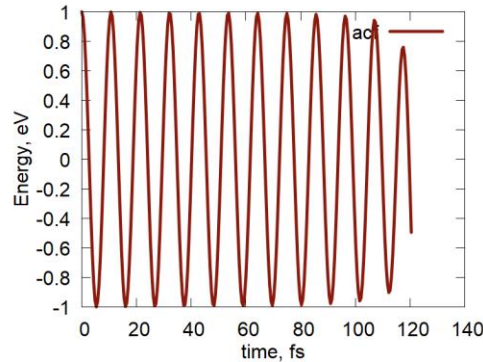
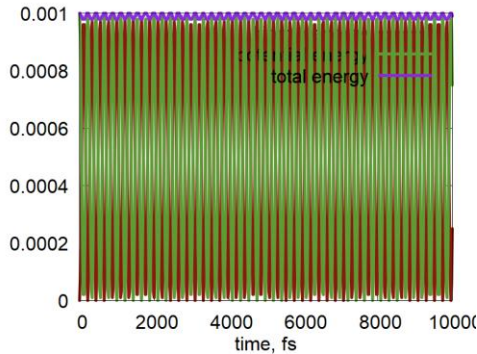
Exercises: ACF and Spectrum of 2-atomic system

In Tut3, prefix = "test1"

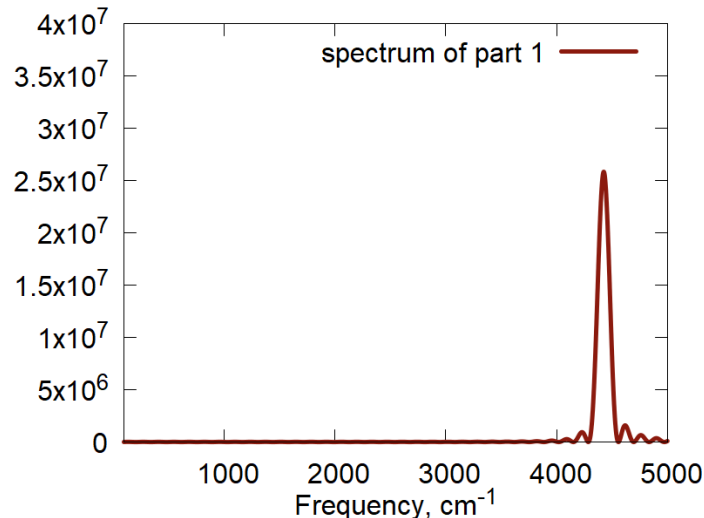
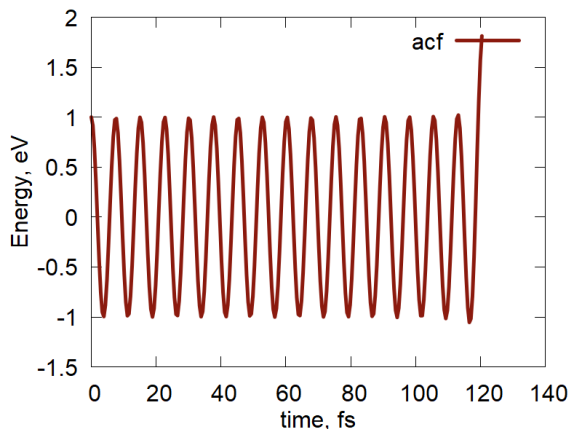
Analytic calculation:

Frequency = $3103.79623215 \text{ cm}^{-1}$

Normal modes give: $3103.79623 \text{ cm}^{-1}$



In Tut3, prefix = "test2"

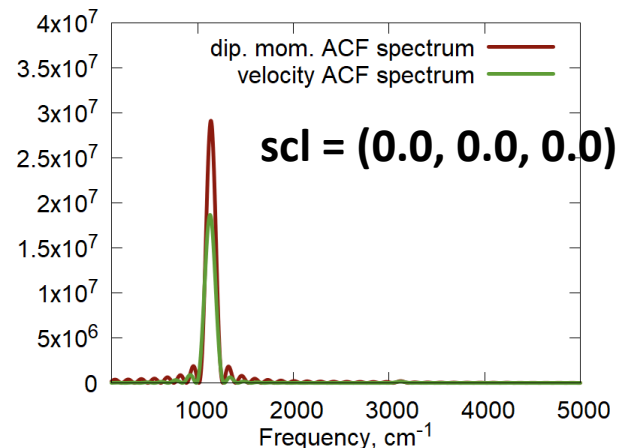
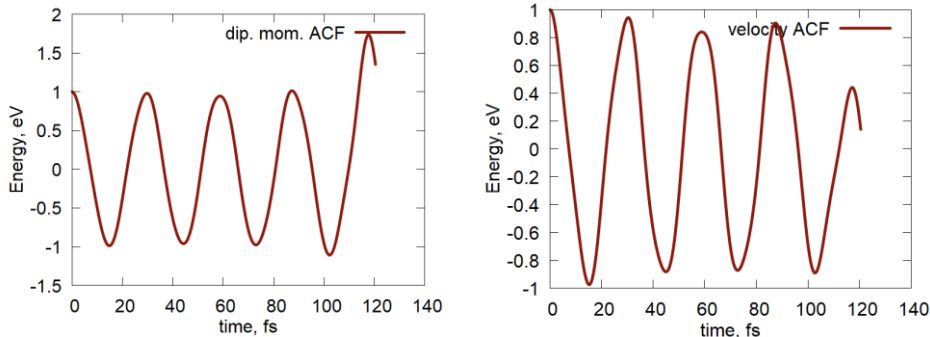


Normal modes give:

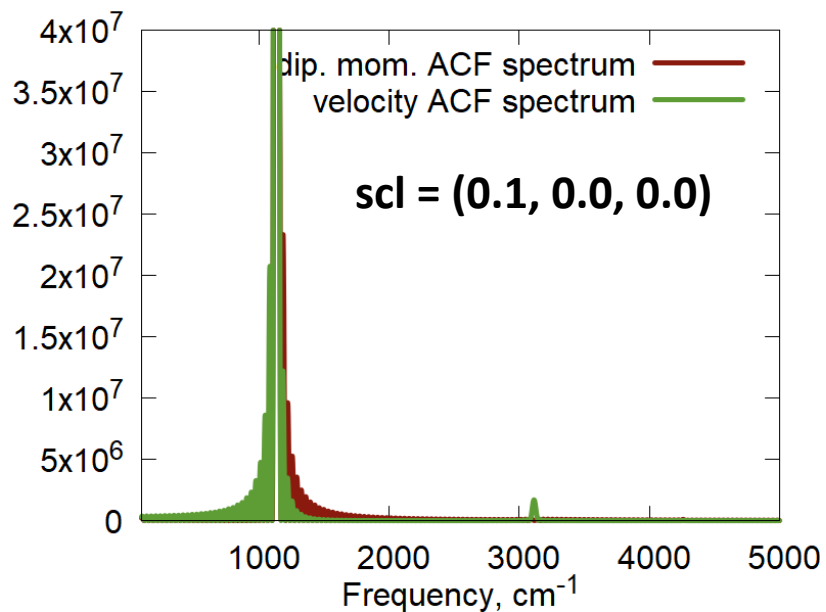
4389.43073

Exercises: ACF and Spectrum of a linear chain

In Tut3, prefix = “test3”



In Tut3, prefix = “test4”,

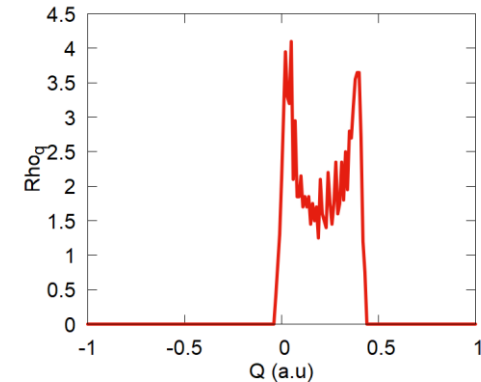
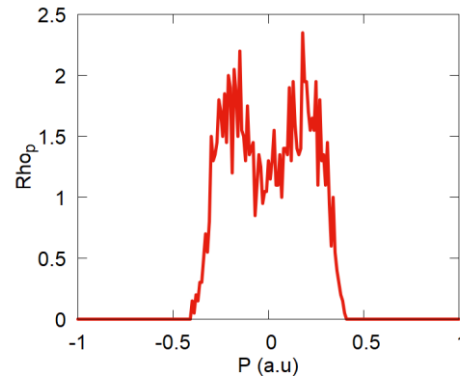
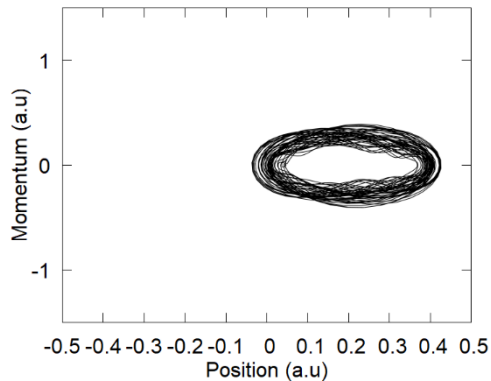
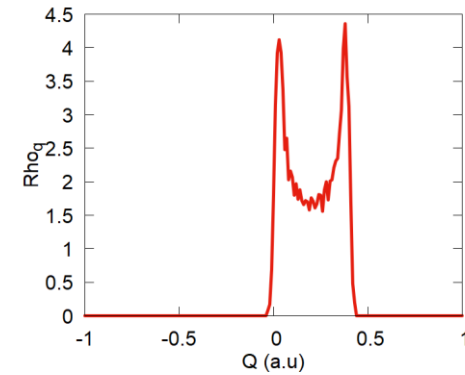
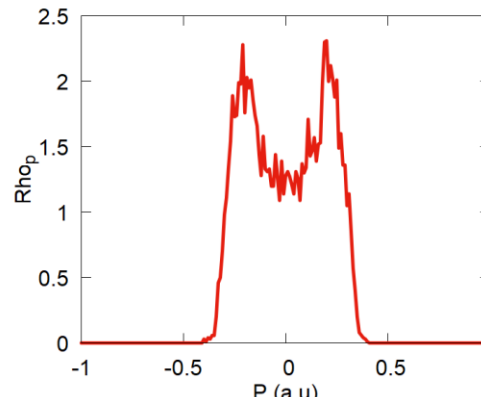
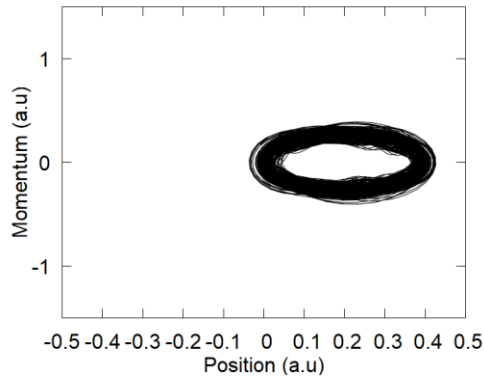
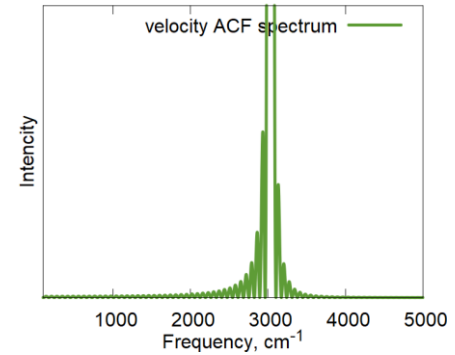
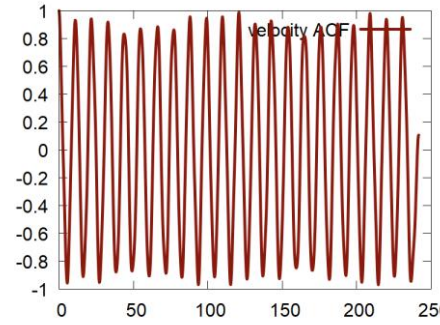
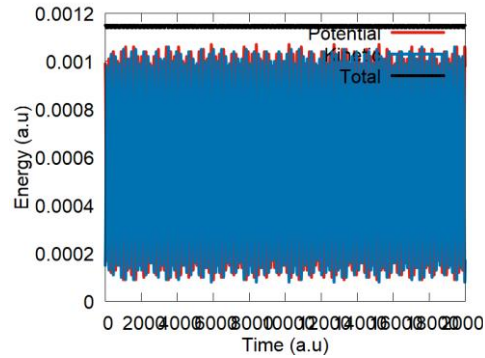
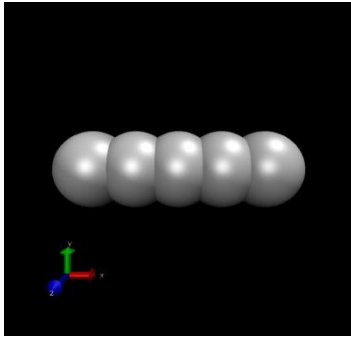


Normal modes give:

- 1136.06827 cm^{-1}
- 2194.71536 cm^{-1}
- 3103.79623 cm^{-1}
- 3801.35852 cm^{-1}
- 4239.86450 cm^{-1}

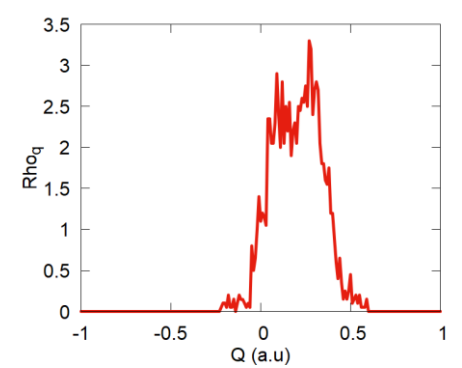
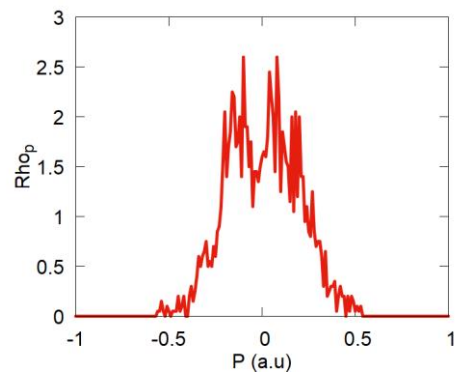
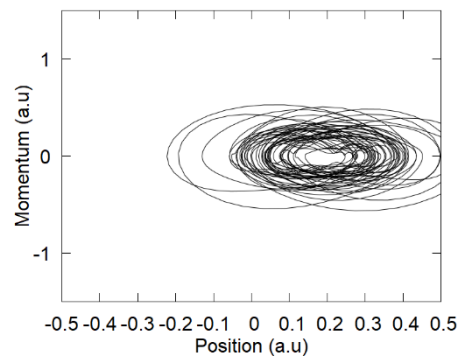
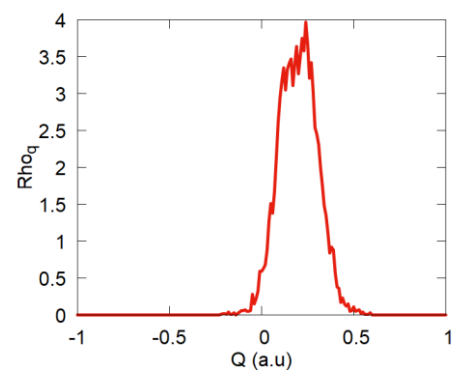
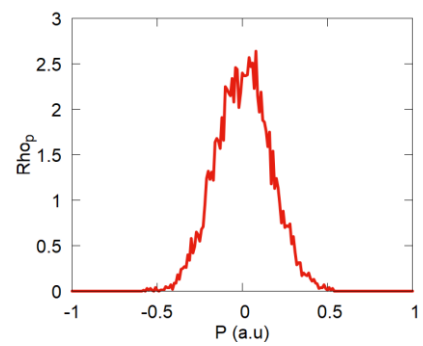
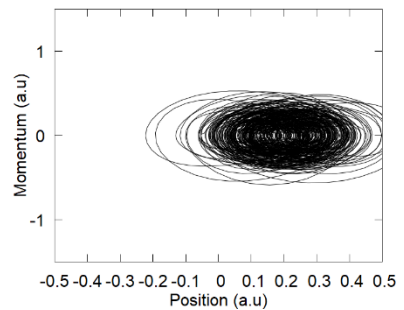
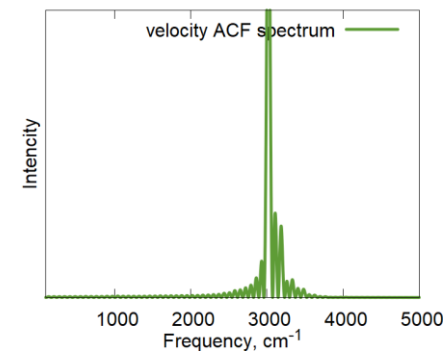
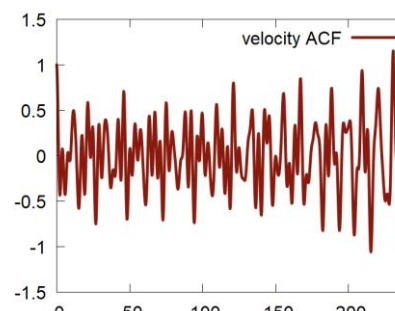
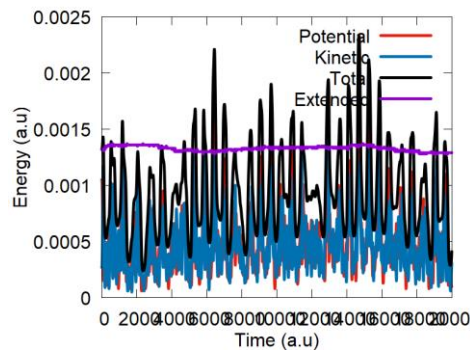
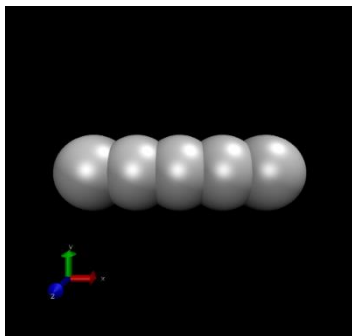
Exercises: Linear chain in NVE ensemble

Tut4: NVE



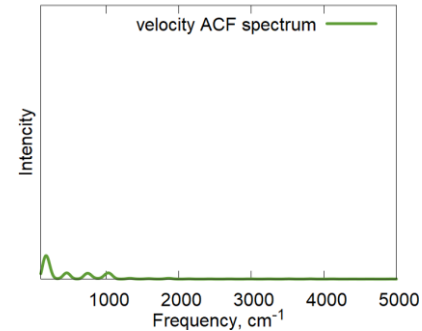
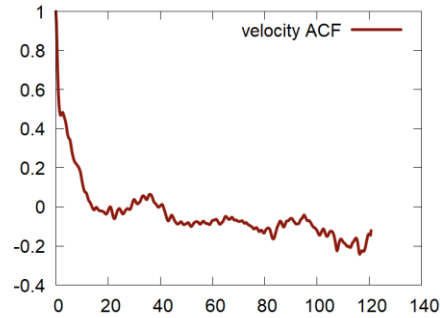
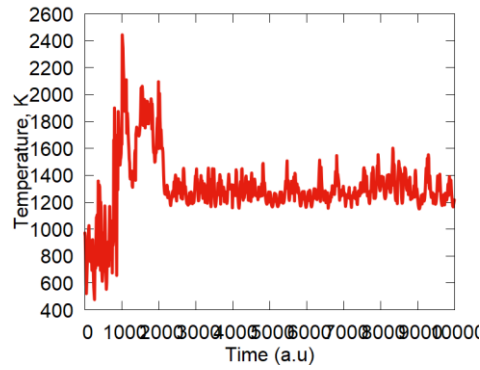
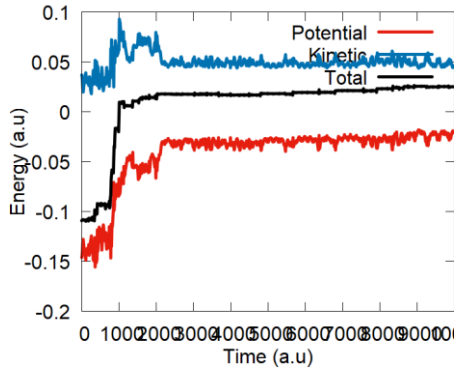
Exercises: Linear chain in NVT ensemble

Tut4: NVT



Exercises: Dynamics of LJ cluster

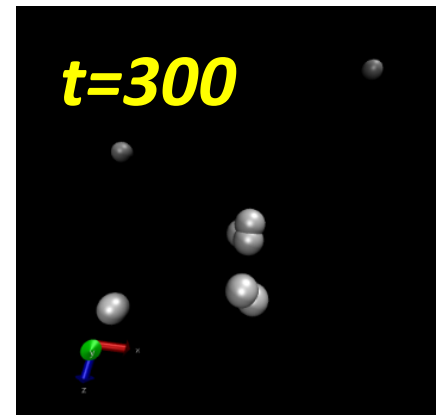
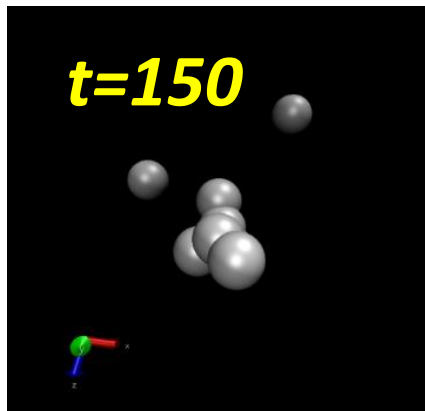
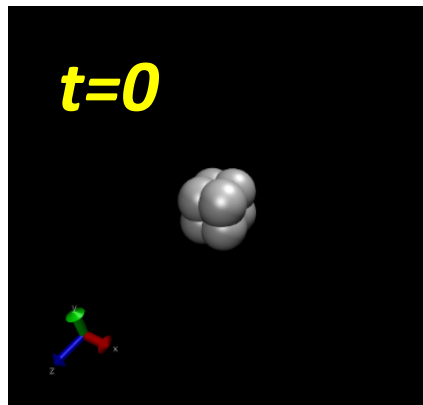
Tut5: NVE of LJ cluster, no thermalization – nve.py



Energy is not conserved!
(phase transitions)

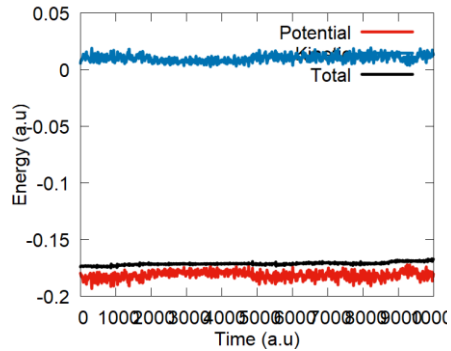
Temperature is high!

Low-frequency modes
are indicative of translational motion

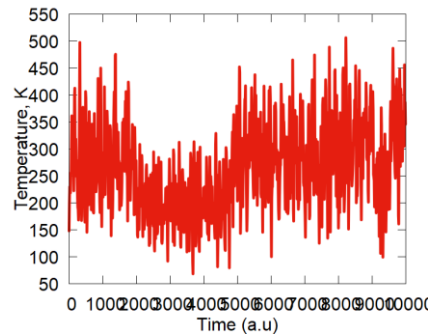


Exercises: Dynamics of LJ cluster

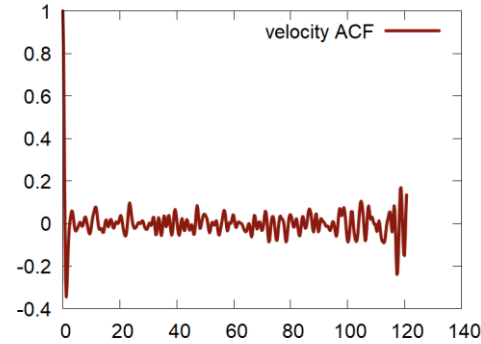
Tut5: NVE of LJ cluster, with thermalization (simulated annealing) – nve2.py



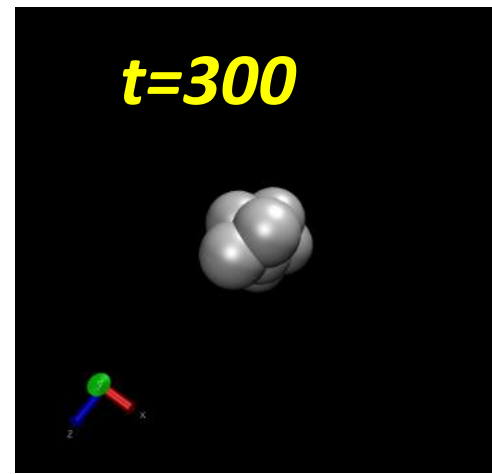
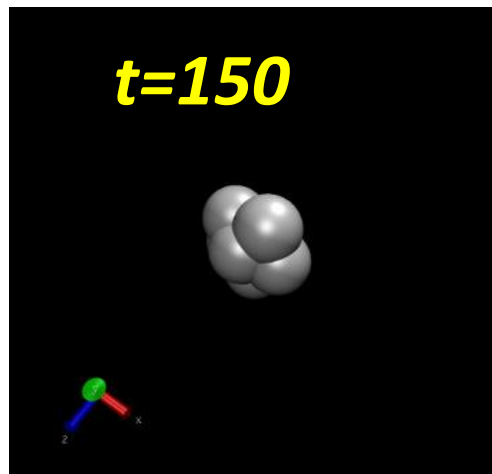
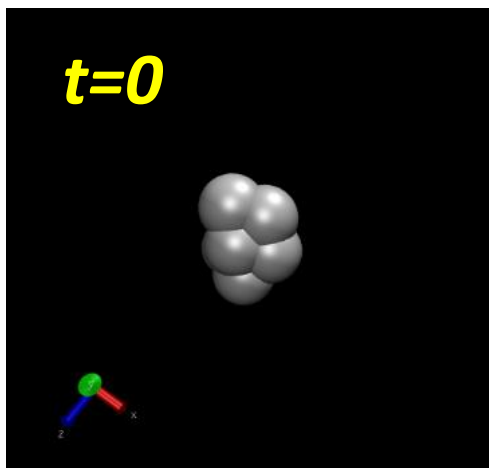
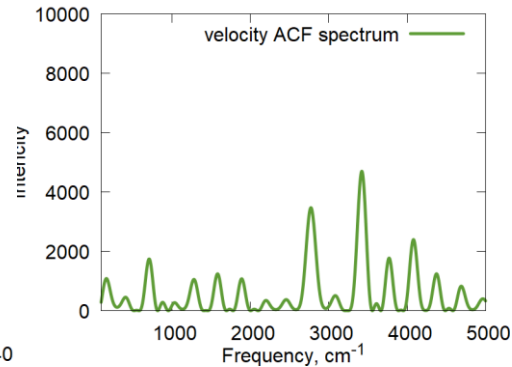
Energy is well conserved!



Temperature is reasonable

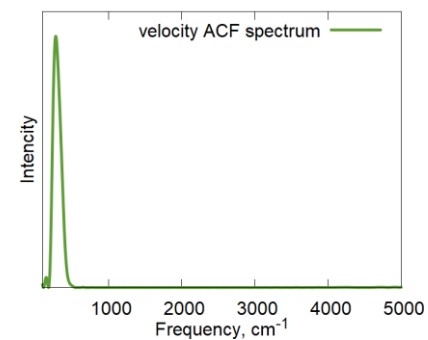
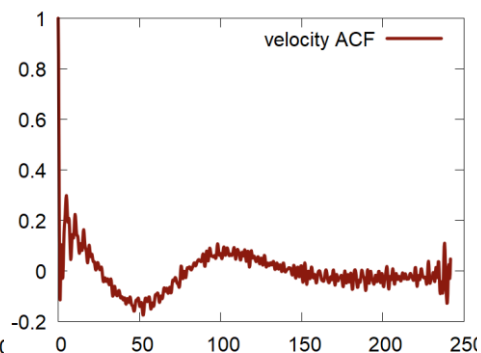
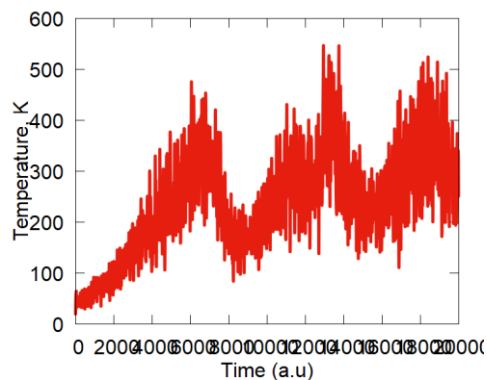
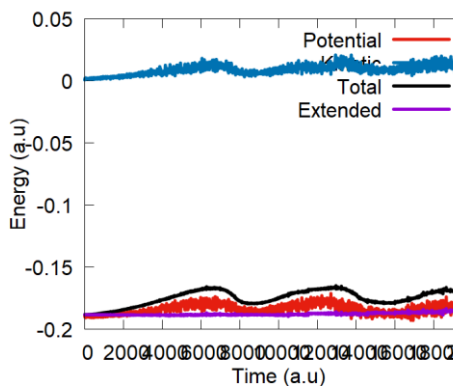


Higher-frequency modes are resolved

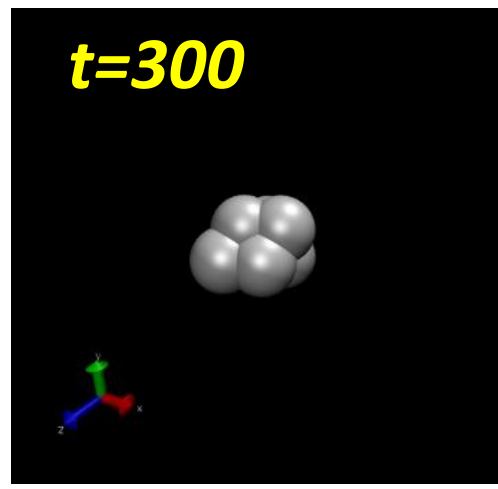
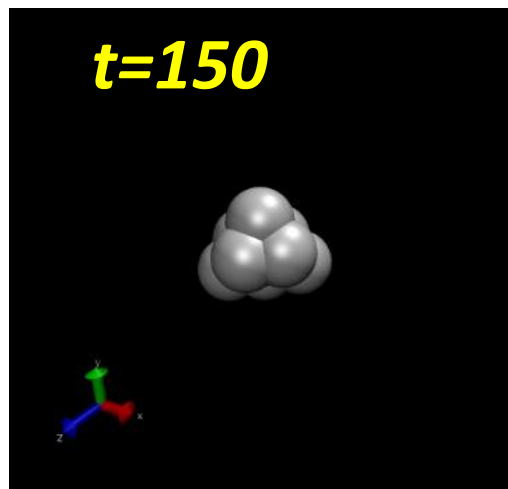
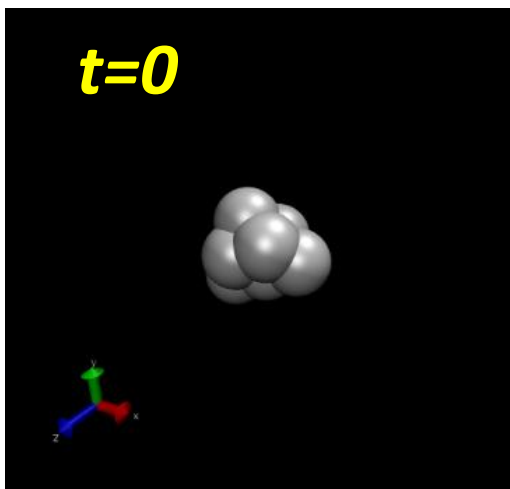


Exercises: Dynamics of LJ cluster

Tut5: NVT of LJ cluster after simulated annealing (nvt.py), slow bath

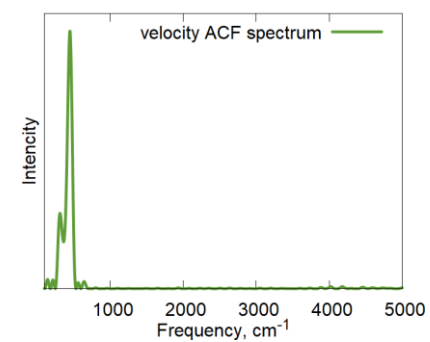
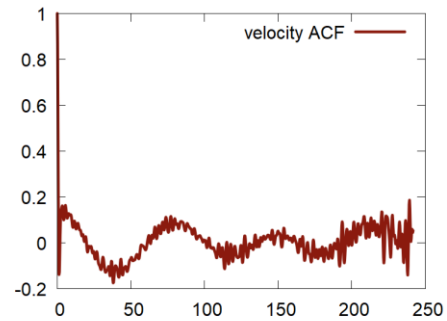
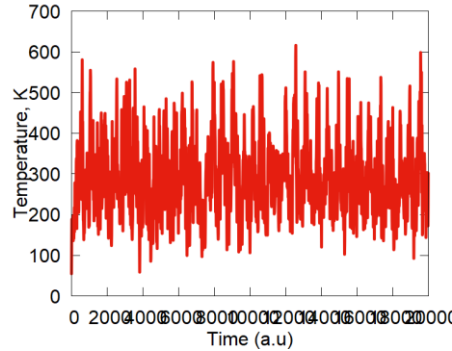
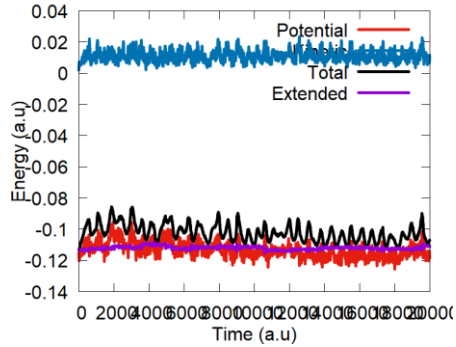


Beware: Bath modes!

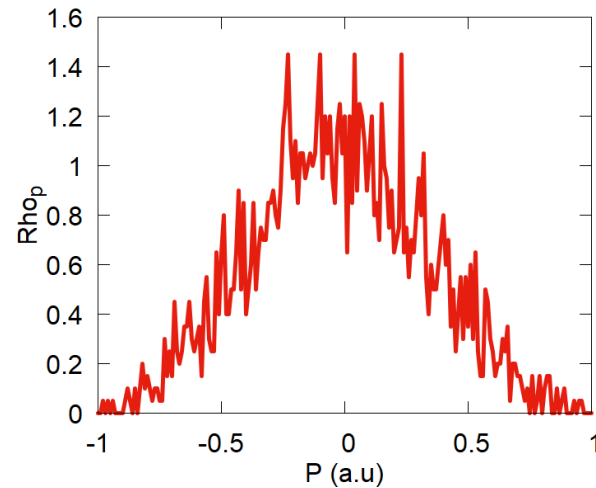
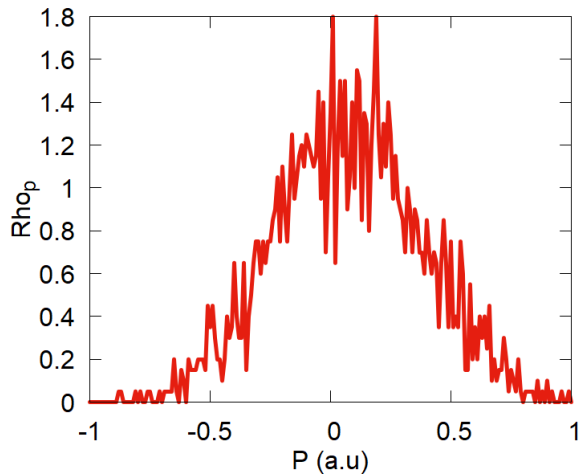


Exercises: Dynamics of LJ cluster

Tut5: NVT of LJ cluster after simulated annealing (nvt.py), fast bath

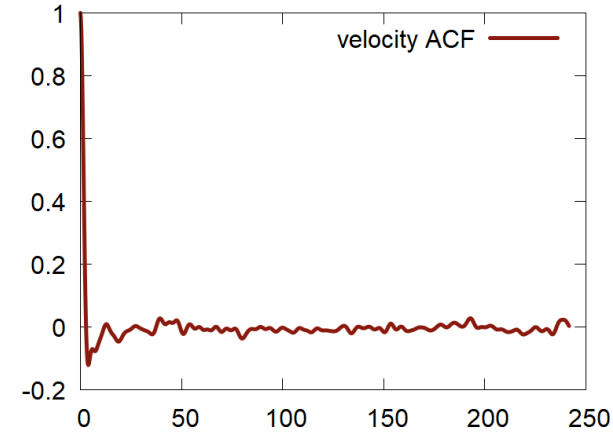
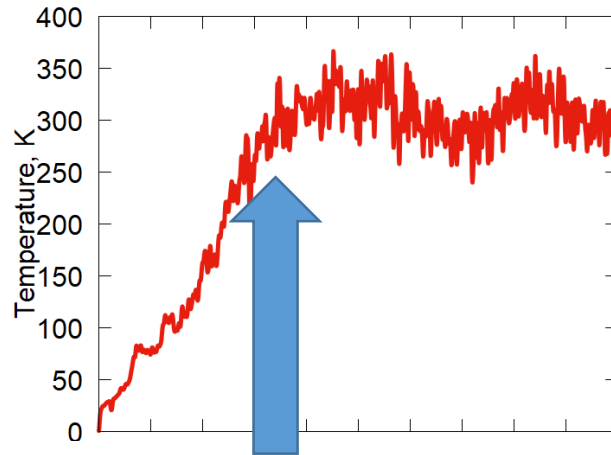
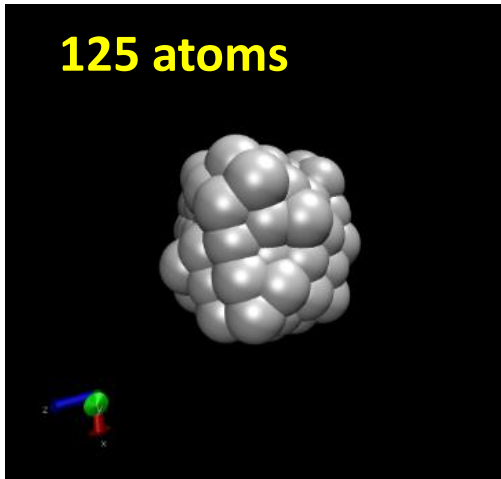


Beware: Bath modes!

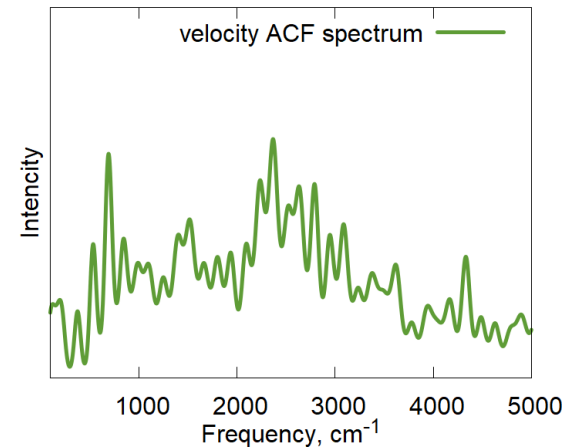
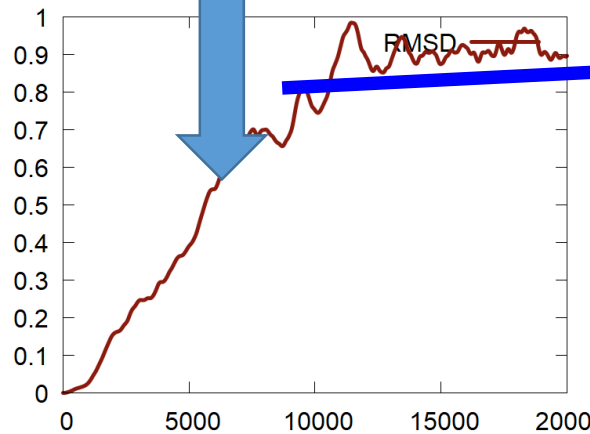
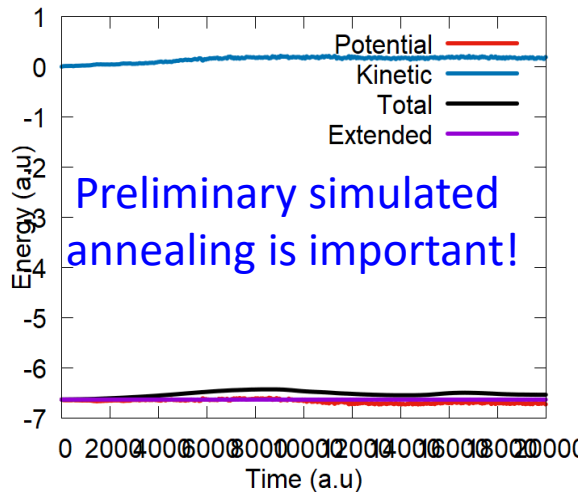


Exercises: Diffusion coefficient, larger cluster

Tut6:



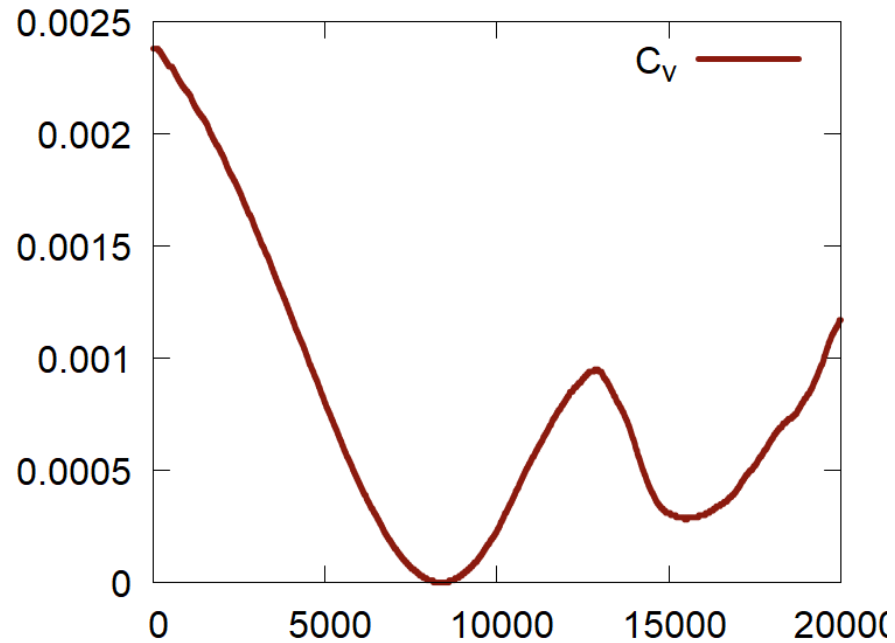
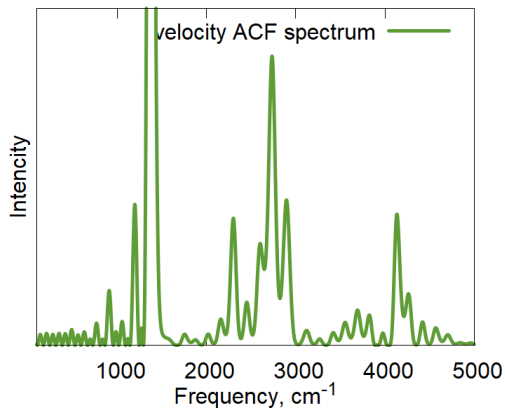
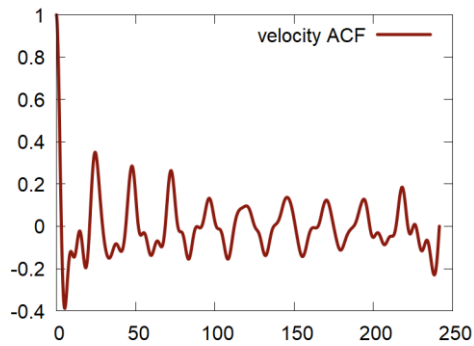
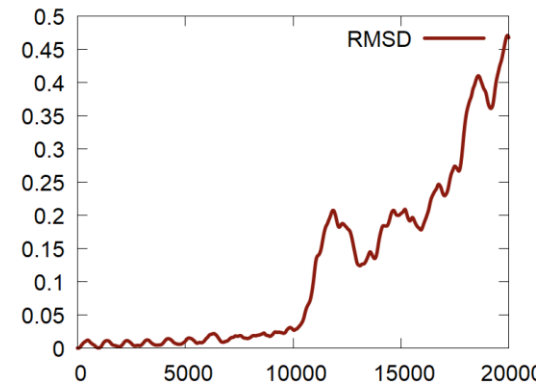
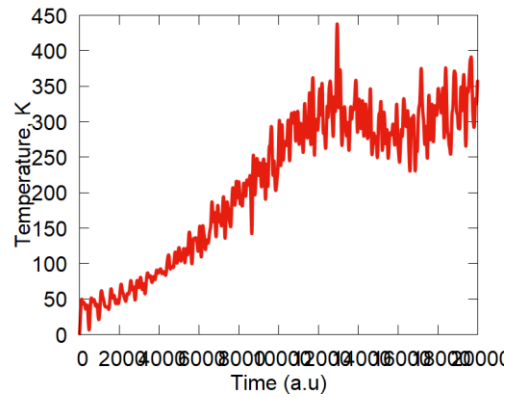
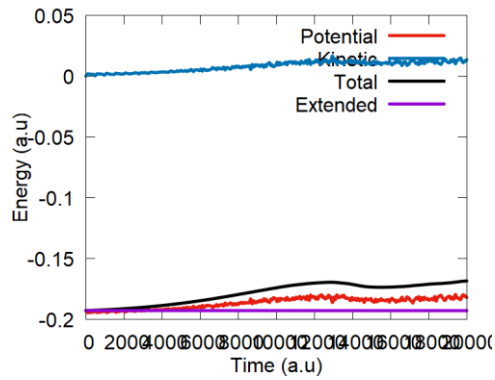
Equilibration period!!!



$$D = \frac{\langle |q(t) - q(0)|^2 \rangle}{6t}$$

Exercises: Heat capacity

Tut7:



Summary of Tutorials

Tut1 – demonstration of MD for N_{traj} trajectories, each with 1 particle. NVE and NVT

Tut2 – demonstration of computing the ACF and its FT for a predefined sequence

Tut3 – computing ACF for a chain of particles connected by springs

Tut4 – going back to MD, for a chain of atoms.

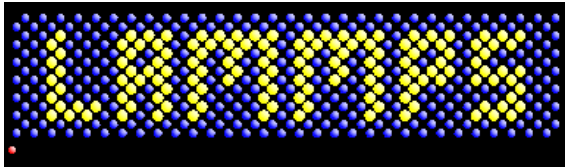
Tut5 – MD of a LJ cluster

Tut6 – MD of a larger LJ cluster, computing diffusion coefficients

Tut7 – MD of a LJ cluster, computing heat capacity

Overview of software

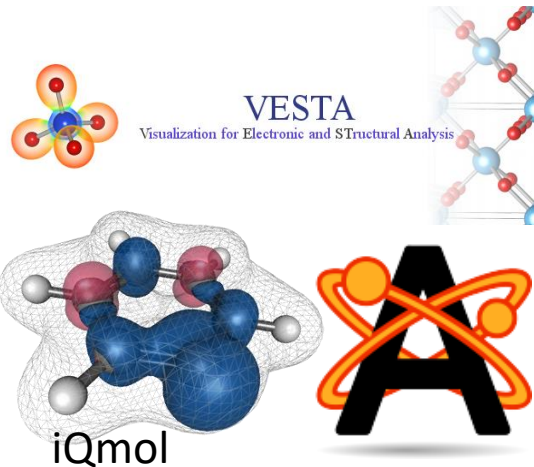
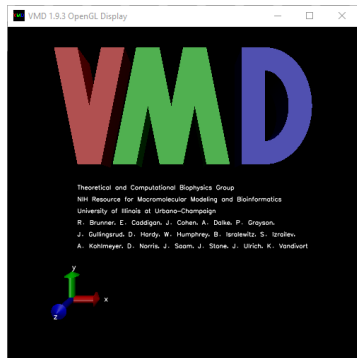
Material Simulations



Biological Systems



Visualization



Quantum Calculations

